淡江大學95學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：高等微積分

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁

1. If $s_1 = \sqrt{2}$ and $s_{n+1} = \sqrt{2 + s_n}$ for $n = 1, 2, 3, \cdots$. Prove that $\{s_n\}$ converges and find $\lim_{n\to\infty} s_n$. (14 points).

2. Let $f: \mathbb{R} \to \mathbb{R}$ be continuous. and for any $x \neq 0$, $f'(x)$ exists. If $\lim_{x\to 0} f'(x) = 2$, show that $f'(0) = 2$. (14 points).

3. Assume that $f(x) \geqq 0$ and that f is continuous and decreases monotonically on $[1, \infty)$, prove that $\int_1^\infty f(x)\,dx$ converges if and only if $\sum_{n=1}^{\infty} f(n)$ converges. (15 points)

4. It is known that $e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$. Show that e is irrational. (14 points)

5. (a) Let $w = x^2 + y^2 + z^2$ and $x = u + v$, $y = u - v$, $z = u \cdot v$. Find $\frac{\partial w}{\partial u}$ when $(u, v) = (1, 2)$.

   (b) If $x^2 + xy + yz + z^3 - 12 = 0$, find $\frac{\partial z}{\partial x}$. (14 points)

6. For $n = 1, 2, 3, \cdots$, x real, put $f_n(x) = \frac{x}{1 + n^2 x^2}$. Show that $\{f_n\}$ converges uniformly to a function f. and that the equation $f'(x) = \lim_{n\to\infty} f_n'(x)$ is correct for $x \neq 0$, but false for $x = 0$. (15 points)

7. Let $f: K \subset \mathbb{R}^n \to \mathbb{R}$ be a continuous function while K is closed and bounded. Show that f has a maximum and a minimum. (14 points.)

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V	簡單型計算機

本試題雙面印製

1. Consider the following linear system

$$\begin{cases} 3x + 4y + 3z = 0 \\ \quad - y - z = 1 \\ x - 2y - 2z = 0 \end{cases}$$

   (a) (5 points) Write down A and b such that the system has the form $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = b$.

   (b) (10 points) Calculate the determinant(行列式) of A, and show that A is invertible(可逆).

   (c) (10 points) Find A^{-1}.

   (d) (5 points) Solve $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ by A^{-1}.

2. (10 points) Suppose that $\{v_1, \cdots, v_n\}$ is a linearly independent(線性獨立) set of vectors in $\mathbb{R}^n$. Let A be a $n \times n$ nonsingular matrix. Show that $\{Av_1, \cdots, Av_n\}$ is linearly independent.

3. Let $A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix}$.

   (a) (10 points) Find the characteristic polynomial(特徵多項式), the real eigenvalues(特徵值), and the corresponding eigenvectors(特徵向量) of A.

   (b) (10 points) Find an invertible matrix C and a diagonal matrix(對角矩陣) D such that $D = C^{-1}AC$.

4. (10 points) Let L_1 and L_2 be linear transformation from a vector space V into a vector space W. Let $\{b_1, \cdots, b_n\}$ be a basis for V. Show that if $L_1(b_i) = L_2(b_i)$ for all i, then $L_1(v) = L_2(v)$ for all $v \in V$.

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5. (10 points) Let P_2 be the set of polynomials with degree less than or equal to 2. Find a basis for the subspace of P_2 consisting of all vectors of the form $ax^2 + bx + 3a - 4b$.

6. (10 points) Use the Gram-Schmidt process to transform the basis $\{(1,1,1),(0,1,1),(1,2,3)\}$ for $\mathbb{R}^3$ into an orthonormal basis for $\mathbb{R}^3$.

7. (10 points) Let $L : P_2 \to P_2$ be the linear transformation defined by

$$L(ax^2 + bx + c) = (a+b)x + b + 2c.$$

   (a) (5 points) Is $-2x^2 + 2x - 4$ in kernal of L?

   (b) (5 points) Is $3x^2 + 1$ in range of L?