

# 淡江大學 95 學年度轉學生招生考試試題

系別：數學學系數學組三年級

科目：代 數

31-1

准帶項目請打「V」

簡單型計算機

本試題共 / 頁

1. Prove that every group of order 5 is abelian. (10%)
2. Suppose that a group  $G$  has subgroups of order 25 and 55.  
If  $200 \leq |G| \leq 300$ , determine  $|G|$ . (10%)
3. Is the converse of the Lagrange's Theorem true? Support your answer. (10%)
4. Let  $\alpha: G_1 \rightarrow G_2$  be a group homomorphism, prove that (20%)
  - (a)  $\alpha(G_1) \leq G_2$  ( $\alpha(G_1)$  is a subgroup of  $G_2$ )
  - (b)  $\ker \alpha \triangleleft G_1$  ( $\ker \alpha$  is a normal subgroup of  $G_1$ )
  - (c)  $\ker \alpha$  is trivial  $\Leftrightarrow \alpha$  is 1-1
5. Characterize cyclic groups. (10%)
6. Show that a group  $G$  is abelian if and only if  $(xy)^2 = x^2y^2$  for all  $x, y$  in  $G$ . (10%)
7. Let  $R$  be a commutative ring with unity 1. Prove that an ideal  $M$  is maximal if and only if  $R/M$  is a field. (10%)
8. Prove that a PID is a UFD but the converse is not true. (20%)