淡江大學九十四學年度轉學生招生考試試題

系別： 數學學系三年級　　　科目：高等微積分

准帶項目請打「V」	
X	簡單型計算機

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1. Show that if $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ cos(nx) converges uniformly on R. (10 points)

2. Use the identity $\frac{1}{1-t}=\sum_{n=0}^{\infty} t^n$, $|t| < 1$, to show that

   $-\ln(1-x) = \sum_{n=0}^{\infty} x^{n+1}/(n+1)$, $|x| < 1$. (10 points)

3. Find $\frac{dy}{dx}$ if $y=\int_0^{\sin x}\sqrt{1+t^2}\,dt$. (10 points)

4. Let $F : R \to R$ be continuous. Show that if F(x)=x for every irrational number x, then F(x)=x for every real number x. (12 points)

5. Let g be dined by $g(x,y)=\frac{xy(y^2-x^2)}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and g(0,0)= 0.

   Find $\frac{\partial^2 g}{\partial x \partial y}(0,0)$ and $\frac{\partial^2 g}{\partial y \partial x}(0,0)$. (12 points)

6. Determine which of $f(x) = \sin(x)$, $g(x) = \sin(1/x)$, $h(x) = x\sin(1/x)$ is uniform continuous on $(0, \infty)$. Give your reason. (18points)

7. Evaluate each of the following :

   (a) $\sum_{n=0}^{\infty} \frac{1}{(n+2)(n+3)}$. (8 points)

   (b) $\lim_{n\to\infty} (1-\frac{x}{n})^n$. (8 points)

8. Let $u(x,y)=x^2-y^2$ and $v(x,y)=2xy$. Compute $\frac{\partial x}{\partial u}$ in term of x and y. (12 points)