

淡江大學九十四學年度轉學生招生考試試題

系別：數學學系數學組三年級 科目：代 數

准帶項目請打「V」

簡單型計算機

節次： 7 月 13 日 第三節

本試題共 1 頁

Show all your work.

10 points each.

1. Show that a group G is abelian if and only if $x^2 = e$ for any x in G , where e is the identity of G .
2. Prove that Z_4 is a field.
3. Let G be a group such that $|G| < 200$. Suppose G has subgroups of order 25 and 35, find the order of G .
4. Let G be the set of all rational numbers except -1 . Show that (G, \oplus) is a group, where $a \oplus b = a + b + ab$ for all a and b in G .
5. Let $G = \langle a \rangle$ be a cyclic group of order 30, where a is a generator of G . Determine $\langle a^5 \rangle$ and $\langle a^2 \rangle$.
6. Up to isomorphism, find all groups of order 4.
7. Show that any group of order 35 is cyclic.
8. Find all distinct subgroups of Z_{12} .
9. Prove that every ideal of the ring of integers is principal.
10. Let G be a group and a and b are in G . Suppose $a^2 = e$ and $ab^4a = b^7$. Show that $b^{33} = e$, where e is the identity of G .