

淡江大學九十三年學年度轉學生招生考試試題 38-1

系別：數學學系三年級

科目：線性代數

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| 准帶項目請打「○」否則打「×」 | |
| ○ | 簡單型計算機 |

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本試題雙面印製

#務必書寫過計算程，否則不予計分。

1. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $T((x, y, z)) = (x, x+y, x+y+2z)$.

Prove that T is invertible(可逆) and find its inverse T^{-1} (T 的逆變換). (10 points)

2. 將行列式 $\begin{vmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{vmatrix}$ 分解成 x 的一次因式連乘積. (10 points)

3. Let A be $m \times n$ and B be $n \times m$ matrices. Prove that if $m < n$, then BA is not invertible(不可逆). (10 points)

4. Let P_2 be the set of all polynomials of degree at most 2 and $B = \{1, x, x^2\}$. Let $T : \mathbb{R}^3 \rightarrow P_2$, be defined by

$$T(a,b,c) = c + ax + (b-a)x^2 \text{ and } D = \{(1,0,0), (0,1,0), (0,1,1)\}.$$

(a) Show that T is a linear transformation. (5 points)

(b) Find the matrix of T corresponding to the ordered bases D and B . (10 points)

(c) Show that T is one to one and onto. (5 points)

淡江大學九十三年學年度轉學生招生考試試題 35-2

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5. Show that if A is a real symmetric matrix, then eigenvectors of A corresponding to distinct eigenvalues are orthogonal. (對稱矩陣不同 eigenvalue 所對應的 eigenvector 是正交). (10 points)

6. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix}$.

- (a) Find the characteristic polynomial of A . (5points)
- (b) Find eigenvalues and eigenvectors of A . (10points)
- (c) Show that A is not diagonalizable. (5points)
- (d) Show that A is invertible and find its inverse A^{-1} . (8points)

7. 已知 A, B 是同階方陣. 下列敘述若是對的證明之, 若是錯的舉例說明之.

- (a) 若 $AB=BA$ 則 $A^3 B^3 = B^3 A^3$. (6 points)
- (b) 若 $A \neq B$ 則 $A^3 \neq B^3$. (6 points)