淡江大學九十三學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：高等微積分

准帶項目請打「○」否則打「x」	
X	簡單型計算機

節次：7 月 14 日第 4 節
本試題共 1 頁

1. Let $f: [0,1] \to [0,1]$ be continuous, show that there is x in $[0,1]$ such that $f(x) = x$. (13 points)

2. Let $f: [0,1] \to \mathbb{R}$ be continuous, show that f is uniformly continuous. (15 points)

3. Let $f: [0,1] \to [0,\infty)$ be continuous, and $\int_0^1 f(x)\, dx = 0$. Show that for any x in $[0,1]$, $f(x) = 0$. (13 points)

4. Evaluate $\int_0^\infty e^{-x^2}\, dx$. (13 points)

5. Let $a_0 = 1$, $a_1 = 1$, $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$.
   Find in details $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n}$. (13 points)

6. Suppose f is defined in a neighborhood of x, and suppose $f''(x)$ exists. Show that $\lim\limits_{h\to 0} \dfrac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$. (13 points)

7. Let $f(x,y) = \begin{cases} \dfrac{x^3 - xy^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

   (a) Show that f is continuous at $(0,0)$

   (b) Find the partial derivatives $f_x(0,0)$, $f_y(0,0)$.

   (c) Find the directional Derivative $D_u f(0,0)$ for a unit vector $u = (u_1, u_2)$

   (d) Is f differentiable at $(0,0)$? (20 points)