系別：數學學系數學組三年級　　　科目：代　　數

准帶項目請打「○」否則打「×」	
×	簡單型計算機

節次：7 月 14 日第 三 節
本試題共　1　頁

Show your work.

1. Show that a group G is abelian if and only if $(xy)^2 = x^2y^2$ for all x, y in G. (10%)

2. Find all distinct subgroups of Z_{18}. (10%)

3. Up to isomorphism, find all groups of order 6. (10%)

4. Show that any group of order 15 is cyclic. (10%)

5. If p is a prime, prove that $a^p - a$ is divisible by p for all integers a. (10%)

6. Let R be a ring with unity 1. If $r^3 = r$ for all r in R, Show that R is commutative. (20%)

7. Let R be a commutative ring with unity 1. Prove that an ideal M is maximal if and only if R/M is a field. (20%)

8. Let f: G → G' be a group homomorphism, prove that ker f is a normal subgroup of G. (10%)