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淡江大學九十二學年度轉學生招生考試試題

系別：數學系三年級　　　　科目：線　性　代　數

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○	簡單型計算機

本試題雙面印製

#務必書寫過計算程，否則不予計分。

1. Let $A = \begin{bmatrix} 1 & -1 & +0 & +0 \\ 0 & +1 & -1 & +0 \\ 0 & +0 & +1 & -1 \\ a & +b & +c & 1+d \end{bmatrix}$. Find det(A) . (10 points)

2. Let T : $R^3 \rightarrow R^2$ be a linear transformation and T(1,0,0)=(1,-2), T(1,1,0)=(1,3), T(0,0,1)=(2,-1). Find T(x,y,z). (10 points)

3. Let $A = \begin{bmatrix} 1 & -1 & 2 & -2 & 3 \\ 2 & -2 & 4 & -4 & 6 \\ 3 & -3 & 6 & -6 & 9 \\ 4 & -5 & 7 & -7 & 9 \end{bmatrix}$ and $X=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$.

   (a) Find general solutions of AX=0. (10 points)

   (b) Find Rank(A). (5points)

4. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$. Show that P is invertible and find P^{-1} .(10 points)

5. Let T : $P_2 \rightarrow R^2$ be defined by T(a+bx+cx^2) = (a-b,c+a) and B={1, x, x^2}, D={(1,-1),(0,1)}. Find the matrix of T corresponding to the ordered bases B and D. (10 points)

注意背面尚有試題▶

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6. Suppose that {x, y} is a linearly independent set in a vector space V. Show that if T : V→ W is a one –to-one linear transformation, then {T(x+2y), T(2x-y)} is also linearly independent. (10 points)

7. Let $A = \begin{bmatrix} 2 & -5 \\ 0 & -1 \end{bmatrix}$.
   (a) Find the characteristic polynomial of A. (5points)
   (b) Find an invertible matrix P such that $P^{-1}AP$ is diagonal. (10 points)

8. Given line L : x=z-y=y-z in R^3, let P(x,y,z) be the orthogonal projection of (x,y,z) on L. Find P(x,y,z). (10 points)

9. Let T : V→ W be a linear transformation, where V and W are finite dimensional vector spaces. Show that T is onto if and only if there exists a linear transformation S : W→ V such that TS(w)= w for all w in W. (10 points)