淡江大學九十二學年度轉學生招生考試試題

系別：數學系三年級　　科目：高 等 微 積 分

准帶項目請打「○」否則打「×」	
X	簡單型計算機

本試題共 1 頁

1. (a) Does $\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{x^2+y^2}$ exists? Explain! (b) Find $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

2. Let $f(x)=\begin{cases} x\sin(\frac{1}{x}) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$, is f continuous at 0? Explain!

3. (a) $y=x^x$, find $\frac{dy}{dx}$, (b) $y=e^x\sin x$, find $\frac{dy}{dx}$.

4. Use <u>definition</u> to find $\int_0^1 x\,dx$.

5. Let $f(x)=\frac{1}{x}$ $\forall x\in(0,1)$. Is f uniformly continuous on $(0,1)$. Explain.

6 (a) Suppose that $a_n, a\in\mathbb{R}$, and $\lim_{n\to\infty} a_n = a$. Prove that

$$\lim_{n\to\infty} \frac{a_1+a_2+\cdots+a_n}{n} = a.$$

(b) Show that the converse of (a) is false

7. In $\mathbb{R}^n$, prove that every closed and bounded set K is compact

8. Suppose f is a continuous mapping of a compact metric space $\overline{X}$ into a metric space Y. Prove that $f(\overline{X})$ is compact

9. State and prove the Inverse Function Theorem.

10. If $\{f_n\}$ is a sequence of continuous functions on $[0,1]$, and if $f_n \to f$ uniformly on $[0,1]$. Prove that f is continuous on $[0,1]$.

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