淡江大學九十一學年度日間部轉學生招生考試試題

系別：數學系三年級　　　　科目：線性代數

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計算機

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1. (20%)

(a) If A is a $m\times n$ matrix and B is a $n\times m$ matrix, show that $tr(AB)=tr(BA)$. where $tr(C)$ is the sum of the main diagonal elements c_{ii}, $i=1,2,\cdots,n$ of a square matrix C of order n.

(b) Let A, B, and O be 2×2 matrices. Assuming that A is invertible, find a matrix C such that C is the inverse of the partitioned matrix $\left[\begin{array}{c|c} A & O \\ \hline B & A \end{array}\right]$

2. (20%)

(a) Let $A=\begin{bmatrix} 2 & -1 & 2 & 5 & 6 \\ 4 & 3 & -1 & 3 & 4 \\ 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & -2 & 6 & 2 \\ 0 & 0 & 3 & 5 & 2 \end{bmatrix}$. Find $\det(A)$.

(b) Let $A=\begin{bmatrix} \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ \cos^2\alpha & \cos^2\beta & \cos^2\gamma \\ 1 & 1 & 1 \end{bmatrix}$. Find $\det(A)$

3. (20%)

(a) Find a basis for the space spanned by the vectors

$V_1=(1,-2,0,0,3)$, $V_2=(2,-5,-3,-2,6)$, $V_3=(0,5,15,10,0)$

$V_4=(2,6,18,8,6)$

(b) Find a standard basis vector that can be added to the set $\{V_1, V_2\}$ to produce a basis for $\mathbb{R}^3$. where

$V_1=(-1,2,3)$, $V_2=(1,-2,-2)$.

◀注意背面尚有試題▶

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4. (20%)

(a) Let $\mathbb{R}^3$ have the Euclidean product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis, where
$u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$, $u_3 = (1, 2, 1)$.

(b) Let $\mathbb{R}^4$ have the Euclidean product and let W be the subspace spanned by $u_1 = (1, 1, 1, 0)$, $u_2 = (-1, 1, 0, 0)$, $u_3 = (1, 2, 1, 0)$. Find the orthogonal projection of $u = (1, 2, 2, 1)$ on W.

5 (20%)

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}$$

and let $B = \{u_1, u_2\}$ be the basis for which $u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(a) Find the matrix for T with respect to B.

(b) Find the matrix for $(T)^n$ with respect to B, where n is a positive integer.