淡江大學九十一學年度日間部轉學生招生考試試題

系別：數學系三年級　　　　　　科目：高等微積分

准帶項目請打「○」否則打「x」
計算機
X

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本試題雙面印製

一、(12%)

(1) Find $f'(x)$, if $f(x)=x^x$ ~~$f'(x)=x^x$~~, $x>0$.

(2) Find $\frac{dy}{dx}$, if $yx^2=\cos x+e^y\ln(1+x^2)$.

二、Evaluate each of the following integrals(12%)

(1) $\int_1^2(\ln x)^3dx$.

(2) $\int\frac{dx}{x^2-1}$.

三、If f is a bounded function on $[a,b]$, and $P=\{a=x_0,x_1,x_2,\dots,x_n=b\}$ is a partition of $[a,b]$. Let $M_i=\sup\{f(x):x\in[x_{i-1},x_i]\}$,

$m_i=\inf\{f(x):x\in[x_{i-1},x_i]\}$, $\Delta x_i=x_i-x_{i-1}$, $i=1,2,\dots,n$

$U(P,f)=\sum_{i=1}^{n}M_i\Delta x_i$, $L(P,f)=\sum_{i=1}^{n}m_i\Delta x_i$.

$\underline{I}=\sup\{L(P,f):P\text{ is a partition of }[a,b]\}$

$\bar{I}=\inf\{U(P,f):P\text{ is a partition of }[a,b]\}$.

We say that f is Riemann integral on $[a,b]$ if $\underline{I}=\bar{I}$, in this case, we write $\int_a^b f(x)dx=\underline{I}=\bar{I}$.

(1) If $Q=P\cup\{c\}$ where $c\in(a,b)$, $c\notin P$, show that

$$U(Q,f)\le U(P,f),$$

$$L(Q,f)\ge L(P,f).\qquad(10\%)$$

(2) If P,Q are any partitions of $[a,b]$, show that

$$L(P,f)\le U(Q,f).\qquad(5\%)$$

(3) Given $\varepsilon>0$, if there is a partition P_ε of $[a,b]$ such that

$$U(P_\varepsilon,f)-L(P_\varepsilon,f)\le\varepsilon,$$

show that f is Riemann integrable on $[a,b]$. (8%)

◀注意背面尚有試題▶

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(4) If f is continuous on $[a, b]$, show that f is Riemann integrable on $[a, b]$. (10%)

四、If $f_n : [a,b] \to R$ is continuous on $[a, b]$, $n = 1,2,\ldots$, and $\lim_{n\to\infty} f_n(x) = f(x)$ uniformly on $[a, b]$, show that f is continuous on $[a, b]$. (14%)

五、If $\{a_n\}$ is a sequence of real numbers, show that $\lim_{n\to\infty} a_n = a$ if, and only if, $\liminf_{n\to\infty} a_n = \limsup_{n\to\infty} a_n = a$. (14%)

六、Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms. Show that

(a) if $\limsup_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, the series converges absolutely.

(b) if $\liminf_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$, the series diverges.

(c) if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, then no conclusion. (15%)