准帶項目請打「○」否則打「x」
計 算 機
×

10 points each.

1. State the definitions of group, ring and field.

2. Show that every subgroup of a cyclic group is cyclic.

3. Find all distinct subgroups of Z_{12}.

4. Show that a group G is abelian if $(ab)^2 = a^2 b^2$ for all a,b in G.

5. Describe all groups of order ≤ 6.

6. Let $\Phi: G \to H$ be a group homomorphism, prove that $\ker\Phi$ is a normal subgroup of G.

7. Let $Q[\sqrt{5}] = \{ a + b\sqrt{5} \mid a, b \in Q\}$, show that $Q[\sqrt{5}]$ is a field under addition and multiplication.

8. Let H and K be subgroups of a group G. If $|H|$ and $|K|$ are relatively prime, prove that $H \cap K = \{e\}$, e is the identity of G.

9. Show that any group of order 35 is abelian.

10. Let R be a commutative ring, prove that $\{ r \in R \mid r^n = 0$ for some positive integer n$\}$ is an ideal of R.