淡江大學九十學年度日間部轉學生招生考試試題

系別：數學系三年級

科目：線 性 代 數

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1. (10%) Determine whether the set of column vectors

$$\mathbf{C}=\left\{\begin{bmatrix}2\\6\\-2\end{bmatrix},\begin{bmatrix}3\\1\\2\end{bmatrix},\begin{bmatrix}8\\16\\-3\end{bmatrix}\right\}$$

is linearly independent.

2. (10%)

$$A=\begin{bmatrix}1&3&1&3\\0&1&1&0\\-3&0&6&-1\\3&4&-2&1\\2&0&-4&-2\end{bmatrix}.$$

(a) Find a basis for the row space of the matrix A.

(b) What is the rank of A?

3. (20%)

(a) Let $A=\begin{bmatrix}1&1&2&-2\\1&5&2&-1\\-2&-2&1&3\\-3&4&-1&8\end{bmatrix}$. Find $detA$.

(b) Let $A=\begin{bmatrix}2&0&-1\\1&1&1\\3&2&-3\end{bmatrix}$. Find $det(A^{-1})$.

4. (10%) Determine whether the given transformations are linear.

(a) $T:\mathbf{R}^3\to\mathbf{R}^2;\ T[a,b,c]=[a+b,c]$

(b) $T:\mathbf{P}^2\to\mathbf{P}^2;\ T(a_2t^2+a_1t+a_0)=(a_2-a_1)t^2+(a_1-a_0)t$

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5. (20%) Let $\mathbf{C} = \{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$ and $\mathbf{D} = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\}$ be bases for $\mathbf{R}^2$; $T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11a+3b \\ -5a-5b \end{bmatrix}$.

   (a) Find the transition matrix $P_{\mathbf{C}}^{\mathbf{D}}$ from $\mathbf{C}$ to $\mathbf{D}$.

   (b) Find the transition matrix $P_{\mathbf{D}}^{\mathbf{C}}$ from $\mathbf{D}$ to $\mathbf{C}$.

   (c) Find the matrix representation of T, $A_{\mathbf{C}}^{\mathbf{C}}$.

   (d) Find the matrix representation of T, $A_{\mathbf{D}}^{\mathbf{D}}$.

   (e) Give an identity to indicate the relation between the following matrices

$$A_{\mathbf{C}}^{\mathbf{C}},\ A_{\mathbf{D}}^{\mathbf{D}},\ P_{\mathbf{C}}^{\mathbf{D}},\ P_{\mathbf{D}}^{\mathbf{C}}.$$

6. (15%) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. Find A^{99}.

7. (15 %) Prove that if A is a square matrix, then

   (a) $B = \frac{A+A^T}{2}$ is a symmetric matrix.

   (b) $C = \frac{A-A^T}{2}$ is a skew-symmetric matrix.

   (c) AA^T is symmetric for any matrix A.