淡江大學九十學年度日間部轉學生招生考試試題

1. (a) Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x\cdot y}{x^2+y^2}$ does not exist.

   (b) Find $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$. (10%)

2. Suppose $f(x) = \begin{cases} x^2\sin(\frac{1}{x^2}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (10%)

   find $f'(0)$ if it exists.

3. Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$, find a function $F(x)$ such that $F''(x) = f(x)$ (10%)

4. Compute $\int_1^4 x^2\, d[x]$, where $[x]$ is the Gauss integer function (10%)

5. (a) $\dfrac{d}{dx}\int_1^{x^3} \sec t\, dt = ?$ (b) $\dfrac{d}{dx} x^x = ?$ where $x > 0$. (10%)

6. Suppose $f: \mathbb{R}^n \to \mathbb{R}^m$ is a given function, define $f'(x)$ (10%)

7. (a) State Inverse function theorem (20%)

   (b) Suppose $f = (f_1, f_2)$, where $f_1(x_1, x_2) = x_1^2$, $f_2(x_1, x_2) = x_2^2$. Thus $f: \mathbb{R}^2 \to \mathbb{R}^2$, explain at which points f is locally one to one and which points f is not locally one to one.

8. (a) State the Implicit Function Theorem. (20%)

   (b) Suppose $f: \mathbb{R}^5 \to \mathbb{R}^2$ is given by $f = (f_1, f_2)$, where
   $f_1(x_1, x_2, y_1, y_2, y_3) = 2e^{x_1} + x_2 y_1 - 4y_2 + 3$,
   $f_2(x_1, x_2, y_1, y_2, y_3) = x_2\cos x_1 - 6x_1 + 2y_1 - y_3$
   if $a = (0,1)$, $b = (3,2,7)$, then $f(a,b) = 0$.
   Compute $[f'(a,b)]$ and $[g'(3,2,7)]$, where g is the implicit function defined in a neighborhood of $(3,2,7)$.