

# 淡江大學九十學年度日間部轉學生招生考試試題

系別：數學系數學組三年級

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科目：代 數

本試題共 1 頁

Answer all questions. Show your work.

1. Show that a group  $G$  is abelian if  $g^2 = e$  for all  $g$  in  $G$ , where  $e$  is the identity of  $G$ . Give an example showing that the converse is false.
2. Show that any group of order 15 is abelian.
3. Find all subgroups of  $Z_{24}$ .
4. Show that  $x^5 + 6x^4 + 12x + 15$  is irreducible in  $Q[x]$ .
5. Describe the ring  $R = Q[x]/\langle x^2 - 2 \rangle$ , where  $\langle x^2 - 2 \rangle$  is the ideal of  $Q[x]$  generated by  $x^2 - 2$ .
6. Show that an ideal of a commutative ring  $R$  is a prime ideal if and only if  $R/I$  is an integral domain.
7. State and prove Lagrange's Theorem.
8. Show that the center of a group  $G$  is a normal subgroup of  $G$ .
9. Let  $G$  be a group of order 6. Show that  $G$  is cyclic or  $G$  is isomorphic to  $S_3$ .
10. Prove or disprove that a ring  $R$  is a PID if and only if  $R$  is a UFD.