

- (10分) 1. Determine whether the set
 $\{(1, 0, 0, -1), (0, 1, 0, -1), (0, 0, 1, -1), (0, 0, 0, 1)\}$
 is a basis for \mathbb{R}^4 .
- (10分) 2. Find A^{-1} if $A = \begin{bmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.
- (10分) 3. Find the projection of $\vec{b} = (3, 1, -7)$ onto $\vec{a} = (4, 0, -3)$.
- (10分) 4. Solve the following system:

$$\begin{cases} 3x_1 - 6x_2 + 3x_4 = 9 \\ -2x_1 + 4x_2 + 2x_3 - x_4 = -11 \\ 4x_1 - 8x_2 + 6x_3 + 7x_4 = -5 \end{cases}$$
- (10分) 5. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $f(\vec{i} + \vec{j}) = \vec{i} - 3\vec{j}$ and $f(-2\vec{i} + 3\vec{j}) = -4\vec{i} + 2\vec{j}$, where $\vec{i} = (1, 0)$, $\vec{j} = (0, 1)$. Find $f(\vec{i})$ and $f(\vec{j})$.
- (20分) 6. Prove that a square matrix A is orthogonally diagonalizable if and only if A is symmetric.
- (30分) 7. Let $A = \begin{bmatrix} 5 & 0 & -8 & 8 \\ 8 & 1 & -6 & 16 \\ -4 & 0 & 9 & -8 \\ -8 & 0 & 16 & -15 \end{bmatrix}$
- ① Find the characteristic polynomial of A .
 - ② Find the eigenvalues of A .
 - ③ Diagonalize A .