# 淡江大學八十九學年度日間部轉學生招生考試試題

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系別：數學系三年級　　　　科目：高等微積分



1. Find the following : (18%)

(i) $\int_0^{\frac{2}{\pi}} (2x\sin\frac{1}{x} - \cos\frac{1}{x})\,dx$　　(ii) $F'(x)$, where $F(x) = \int_0^1 \ln(t^2+x^2)\,dt$ for $x \neq 0$.

(iii) A series expansion in powers of $x$ of the function

$$f(x) = \int_0^1 \frac{1-e^{-tx}}{t}\,dt, \quad x \in \mathbb{R}.$$

2. Let $\{a_n\}$ be a real sequence and $a \in \mathbb{R}$.

(i) Show that if $a_n \to a$ as $n \to \infty$, then

$$\frac{a_1+a_2+\cdots+a_n}{n} \to a \quad \text{as } n \to \infty. \qquad (16\%)$$

(ii) Show that the converse of (i) is false.

3. Show that if $f:[a,b] \to \mathbb{R}$ is continuous, then $f$ is (Riemann) integrable on $[a,b]$. (16%)

4. Suppos that $f$ is differentiable on $(a,b)$, continuous on $[a,b]$, and $f(a)=f(b)=0$. Show that if $f(c)>0$ for some $c \in (a,b)$, then there exist $x_1, x_2 \in (a,b)$ such that $f'(x_1) > f'(x_2)$. (16%)

5. (i) Show that the series $\sum_{n=1}^{\infty} \frac{\sin(2n+1)x}{(2n-1)(2n)}$ converges uniformly on $[0, \frac{\pi}{2}]$.

(ii) Find a series for $\int_0^{\frac{\pi}{2}} f(x)\,dx$, where $f(x)$ is the sum function of the series in (i).

(16%)

6. Let

$$f(x,y) = \begin{cases} 1 & \text{if } 0<y<x^2. \\ 0 & \text{otherwise}. \end{cases}$$

(i) Show that the directional derivatives of $f$ exists at $(0,0)$ in all directions $\mathbf{u}$.

(ii) Is $f$ continuous at $(0,0)$? Justify your answer.

(iii) Is $f$ differentiable at $(0,0)$? Justify your answer. (18%)