

淡江大學八十八學年度日間部轉學生招生考試試題 27

系別：數學系三年級

科目：線性代數

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Show your work 每題 10 分

1. Solve the system $\begin{cases} x+y-2z+4w=5 \\ 2x+2y-3z+w=3 \\ 3x+3y-4z-2w=1 \end{cases}$ by use of the augmented matrix.
2. Find A^{-1} if $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix}$.
3. Find the rank of $\begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$.
4. Determine whether $(1, 2, -3), (1, -3, 2), (2, -1, 5)$ are linearly dependent.
5. Find a basis for the null space of $A = \begin{pmatrix} 1 & -1 & 2 & 3 & 0 \\ -1 & 0 & -4 & 3 & -1 \\ 2 & -1 & 6 & 0 & 1 \\ -1 & 2 & 0 & -1 & 1 \end{pmatrix}$. What is the nullity of A ?
6. Let V, W be vector spaces. If $T: V \rightarrow W$ is a linear transformation, prove that T is 1-1 if and only if $\ker T = \{\vec{0}\}$.
7. Show that an $n \times n$ matrix A has 0 as an eigenvalue if and only if $\text{Null}(A) \neq \{\vec{0}\}$.
8. If A and B are similar $n \times n$ matrices, prove that A and B have the same eigenvalues.
9. Diagonalize $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -4 & 2 \\ 0 & 0 & -2 \end{pmatrix}$, if possible.
10. Find the orthogonal projection of $\vec{u} = (1, 1, 1)$ onto $\text{Span}\{(1, -1, 1), (-2, 3, -1)\}$.