淡江大學八十八學年度日間部轉學生招生考試試題

系別：數學系三年級　　　　科目：高等微積分

1. (a) If a, b are real numbers such that

$$a \le b + \varepsilon \quad \text{for all } \varepsilon > 0,$$

show that $a \le b$.

(b) If $0 \le a \le \varepsilon$ for all $\varepsilon > 0$, show that $a = 0$. (12%)

2. Suppose that the terms $\{a_n\}$ satisfy

$$|a_{n+1} - a_n| < 2^{-n} \quad \text{for all } n.$$

Prove that $\{a_n\}$ is a Cauchy sequence. (12%)

3. If $f: R \to R$ is continuous at c and $f(c) < 0$, show that there exists a $\delta > 0$ such that

$$f(x) < 0 \quad \text{for all } x \in (c-\delta, c+\delta).$$ (12%)

4. If $f: [a,b] \to [0, \infty)$ is continuous on $[a,b]$ and $\int_a^b f(x)\,dx = 0$, show that $f(x) = 0$ for all $x \in [a,b]$. (12%)

5. (a) State and prove Rolle's Theorem. (12%)

(b) State and prove Mean Value Theorem. (12%)

(c) Prove that $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$. (6%)

6. (a) Suppose that $\lim_{n \to \infty} \sqrt[n]{|a_n|} = \rho$. Show that

(i) if $\rho < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely. (8%)

(ii) if $\rho > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges. (6%)

(b) Test for convergence of $\sum_{n=1}^{\infty} \frac{1}{2^{n+(-1)^n}}$. (8%)