淡江大學八十七學年度日間部轉學生入學考試試題

系別：數學系三年級　　科目：線性代數

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(1) 20% Let $A=\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$. Find (i) determinant $|A|$ (ii) A^{-1}.

(2) 16% Determine whether or not the vectors $(1,-2,1)$, $(2,1,-1)$ $(7,-4,1)$ are linearly dependent.

(3) 16% Consider the homogeneous system $AX=0$, where A is an $m\times n$ matrix. Let W be the subset of R^n consisting of all solutions to the homogeneous system. (W 是所有滿足 $AX=0$ 的解的集合) Show that W is a subspace of R^n.

(4) 16% Let A, B be arbitrary matrices for which the product AB is defined. Show that $\text{rank}(AB) \le \text{rank}(B)$ and $\text{rank}(AB) \le \text{rank}(A)$.

(5) 16% Let $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$. find a nonsingular matrix P such that $P^{-1}AP$ is diagonal.

(6) 16% Suppose U and W are subspaces of a vector space V such that $U=\text{span}(S)$ and $W=\text{span}(T)$. Show that $U+W=\text{span}(S\cup T)$, where $U+W=\{u+w;\ u\in U,\ w\in W\}$

$S\cup T$：聯集