

淡江大學八十七學年度日間部轉學生入學考試試題

系別：數學系數學組三年級

科目：高等代數

本試題共 1 頁

Answer all questions . Show all work.

1. Let $A(T)$ be the group of permutations of the set T and let T_1 be a nonempty subset of T . Prove that $H = \{f \in A(T) | f(t) = t \text{ for all } t \in T_1\}$ is a subgroup of $A(T)$. (11%)
2. If $f : G \rightarrow H$ is an injective homomorphism of groups and $a \in G$, prove that $|f(a)| = |a|$, (11%)
3. If K and N are normal subgroups of a group G such that $K \cap N = \{e\}$, prove that $kn = nk$ for every $n \in N, k \in K$. (11%)
4. If F is a field, show that $F[x]$ is not a field. (11%)
5. Show that $x^5 - 4x + 22$ is irreducible in $\mathbb{Q}[x]$. (11%)
6. (a) Verify that $\mathbb{Q}(\sqrt{3}) = \{r + s\sqrt{3} | r, s \in \mathbb{Q}\}$ is a subfield of \mathbb{R} .
 (b) Show that $\mathbb{Q}(\sqrt{3})$ is isomorphic to $\mathbb{Q}[x]/(x^2 - 3)$. (17%)
7. Let I be an ideal in a ring R . If $K = \{a \in R | ra \in I \text{ for every } r \in R\}$ Prove that K is an ideal. (11%)
8. If R is a ring, write $\langle x \rangle = R[x]x$. Show that $\langle x \rangle$ is an ideal of $R[x]$ and $R[x]/\langle x \rangle$ is isomorphic to R . (17%)