淡江大學104學年度日間部轉學生招生考試試題

系別：物理系三年級 科目：應用數學

考試日期：7月26日 (星期日) 第4節 本試題共5大題, 1頁

1. Consider the Hermitian matrix $A = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix}$.

   (a) Find the eigenvalues and the corresponding eigenvectors of A. [12%]

   (b) Find the inverse matrix of A. [8%]

2. For the Cartesian vector field $\mathbf{a} = (x^2(y+z), y^2(z+x), z^2(x+y))$, evaluate respectively (i) $\nabla\cdot\mathbf{a}$, (ii) $\nabla(\nabla\cdot\mathbf{a})$, (iii) $\nabla^2\mathbf{a}$, (iv) $\nabla\times\mathbf{a}$, and (v) $\nabla\times(\nabla\times\mathbf{a})$. [20%]

3. Consider the following linear differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}.$$

   (a) Find $y_c(x)$ (the complementary solution, i.e., the general solution of $y'' + 2y' + y = 0$). [8%]

   (b) Find the complete solution of this equation with the associated boundary conditions: $y(0) = 1$ and $y'(0) = 0$. [12%]

4. Consider the Fourier representation of a non-periodic function given by $f(x) = x,\ \ 0 \le x < 1$. It can be made periodic with period 2 by continuing this function outside the range as either an even function

$$f_e(x) = \begin{cases} f(x), & 0 \le x < 1, \\ f_e(-x) = f_e(x+2), & \text{for all } x, \end{cases}$$

or an odd function

$$f_o(x) = \begin{cases} f(x), & 0 \le x < 1, \\ -f_o(-x) = f_o(x+2), & \text{for all } x. \end{cases}$$

   (a) Find the Fourier cosine series for $f_e(x)$. [8%]

   (b) Find the Fourier sine series for $f_o(x)$. [8%]

   (c) Which of these two series will be better for numerical evaluation? (State your reasoning based on the relevant periodic continuations or the convergence rate of the Fourier coefficents.) [4%]

5. The generating functions for the Chebyshev polynomials of the first kind and for the Hermite polynomials are, respectively, given by $G_1(x,h) = \frac{1-xh}{1-2xh+h^2} = \sum_{n=0}^{\infty} T_n(x)h^n$ and $G_2(x,h) = e^{2xh-h^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!}h^n$ for $|h| < 1$.

   (a) Find the functions $T_0(x)$, $T_1(x)$, $T_2(x)$, and $T_3(x)$. [1+2+3+4%]

   (b) Find the functions $H_0(x)$, $H_1(x)$, $H_2(x)$, and $H_3(x)$. [1+2+3+4%]