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淡江大學 103 學年度日間部轉學生招生考試試題

系別：物理學系三年級　　科目：理 論 力 學

考試日期：7 月 20 日(星期日) 第 3 節　　本試題共 5 大題， 2 頁

請在 5 題中任選 4 題，每題 25 分

1. A particle moves in a planar orbit described by the position vector $\mathbf{r}=c\cos(\omega t+\alpha)\mathbf{i}+2c\sin(\omega t+\beta)\mathbf{j}$, where c, ω and α are constants, **i** and **j** are the unit vectors along the x- and y-directions, respectively.
   (a) [20%] Find the velocity **v**, the acceleration **a** and the speed of the particle.
   (b) [5%] What is the angle between **v** and **a** at time $t=(\pi/2-\alpha)/\omega$?

2. An undamped harmonic oscillator satisfies the equation of motion $m\frac{d^2x}{dt^2}=-kx+F(t)$, where m and k are the mass and spring constant of the oscillator, respectively.
   (a) [10%]When $F(t)=0$, show that $x(t)=a\sin(\omega_0 t)+b\cos(\omega_0 t)$ is the solution of the equation of motion, where a, b and ω_0 are constants, and find ω_0 (in terms of m and k).
   (b) [10%]A driving force $F(t)=F_0\sin(\omega t)$ is switched on at $t=0$, where F_0 and ω are constants. Find $x(t)$ for $t>0$ with the initial conditions $x=0$ and $v=0$ at $t=0$.
   (c) [5%]Find $x(t)$ for $\omega=\omega_0$ by taking the limit $\omega\to\omega_0$ in your result from part (b).
   Hint: In part (b) you can find a particular solution of the form $x=A\sin(\omega t)$ and determine A.

3. An one-dimensional particle at rest is attracted toward a center by a force $F=-2mk^2/x^3$, where m is the mass of the particle, k is a constant and x is the coordinate of the particle,
   (a) [10%] Find the potential energy resulted from the force.
   (b) [5%] Write down the Newton equation of motion for the particle.
   (c) [10%] Show that the time required for the particle to reach $x=0$ from a distance d is $\frac{d^2}{\sqrt{2}k}$.

4. A system is composed of n particles, with each particle's mass described by m_i, where $i=1, 2, \cdots, n$. The total mass of the system is denoted by M. Show that
   (a) [10%] The linear momentum of the system is the same as if a single particle of mass M were located at the position of the center of mass and moving in the manner the center of mass moves.
   (b) [10%] The time differential of the linear momentum of the system is equal to the sum of all the external forces, as long as the internal forces follow $\mathbf{f}_{ij}=-\mathbf{f}_{ji}$, where $\mathbf{f}_{ij}$ is the force acted on particle i by particle j.
   (c) [5%] The total linear momentum for a system free of external forces is constant and equal to the linear momentum of the center of mass.

5. Two particles of mass m_1 and m_2 move in a plane and interact with each other by a central forces with the potential energy $U(r)=1/2\ k\ r^2$, where k is a constant, $r=|\mathbf{r}_1-\mathbf{r}_2|$, $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors of the two particles.
   (a) [5%] Write down the Lagrangian of the system.
   (b) [10%] Show that the center of mass of the system moves in a constant velocity.

本試題雙面印刷

背面尚有試題

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(c) [10%] Using Lagrangian dynamics to show that the motion of the system can be reduced to an equivalent one-body problem.