

淡江大學 102 學年度日間部轉學生招生考試試題

系別：物理學系三年級

科目：應用數學

考試日期：7月24日(星期三) 第4節

本試題共 5 大題， / 頁

※ 每大題 20 分！滿分 100 分！

1. Solve the set of equations

$$\begin{cases} y' - 2y + z = 0 \\ z' - y - 2z = 0 \end{cases}$$

Subject to the conditions $y(0) = 1$ and $z(0) = 0$.

2. Find the solution of equation $y'' - 2y' + y = 0$ by the following different methods.

(a) Successive integration [連續積分] of two first-order equations.

(b) Power series expansion.

3. (a) $\Phi(x, h) = (1 - 2xh + h^2)^{-1/2} \equiv \sum_{\ell=0}^{\infty} P_{\ell}(x)h^{\ell}$, where $|h| < 1$;

Find the functions $P_0(x) = ?$, $P_1(x) = ?$, $P_2(x) = ?$ and $P_3(x) = ?$

(b) $\Phi(x, h) = e^{(1/2)x(h-h^{-1})} \equiv \sum_{n=-\infty}^{\infty} J_n(x)h^n$;

Find the functions $J_0(x) = ?$, $J_1(x) = ?$, $J_{-1}(x) = ?$ and $J_2(x) = ?$

4. $g(x)$ is a periodic function of 2. [參考下面積分公式]

$$g(x) = x, \quad -1 < x < 1$$

(a) Sketch [描繪] the given functions $g(x)$ on $(-3, 3)$.

(b) Expand $g(x)$ in a sine-cosine Fourier series. [四項之和]

(c) Expand $g(x)$ in a complex exponential Fourier series. [六項之和]

5. (a) Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx = ?$

(b) Evaluate $\int_0^{\infty} e^{-x^2} dx = ?$

$$\left[\begin{aligned} \int x \cos ax dx &= \frac{x \sin ax}{a} + \frac{\cos ax}{a^2} & ; & \int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \\ \int x^2 \cos ax dx &= \frac{1}{a} [x^2 \sin ax] - \frac{2}{a} \int x \sin ax dx & ; & \int x^2 \sin ax dx = -\frac{1}{a} [x^2 \cos ax] + \frac{2}{a} \int x \cos ax dx \end{aligned} \right]$$