※ 每大題 20 分！滿分 100 分！

1. Solve the set of equations

$$\begin{cases} y' - 2y + z = 0 \\ z' - y - 2z = 0 \end{cases}$$

subject to the initial conditions $y_0 = 1$, $z_0 = 0$.

2. Find the solution of the following differential equations.

   (a) $y'' + 2y' - 3y = e^x$

   (b) $y'' + 4y = \sin 2x$

3. Find the eigenvalues and corresponding eigenvectors of the following matrices.

   (a) $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$　　(b) $B = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

4. (a) Expand the periodic function in a sine-cosine Fourier series.

   $$F(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

   (b) Qualitatively sketch a graph of the sum of three terms of the series.
   [Hint: Sketch each term separately on the same axes and add the terms graphically.]

5. (a) Evaluate $\int_0^\infty \dfrac{\sin x}{x}\,dx = ?$

   (b) Evaluate $\int_0^\infty e^{-x} x^n \, dx = ?$ for $n = 1, 2,$ and 3.