

淡江大學 99 學年度轉學生招生考試試題

系別：物理學系三年級

科目：理 論 力 學

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1. A particle moves in a planar elliptical orbit described by the position vector $\mathbf{r} = 2b \sin(\omega t + \beta)\mathbf{i} + b \cos(\omega t + \beta)\mathbf{j}$, where b and β are constants, \mathbf{i} and \mathbf{j} are the unit vectors along the x - and y -directions, respectively. We also use \mathbf{v} and \mathbf{a} to denote the velocity and acceleration of the particle.

(a) [12%] Find \mathbf{v} , \mathbf{a} , and the particle speed.

(b) [3%] What is the angle between \mathbf{v} and \mathbf{a} at time $t = \left(\frac{\pi}{2} - \beta\right) / \omega$?

(c) [10%] Show that in general

$$\frac{d}{dt} [\mathbf{a} \cdot (\mathbf{v} \times \mathbf{r})] = \dot{\mathbf{a}} \cdot (\mathbf{v} \times \mathbf{r}), \quad \text{where } \dot{\mathbf{a}} = \frac{d\mathbf{a}}{dt}.$$

2. An one-dimensional particle at rest is attracted toward a center of force with potential energy $U = -mk^2/(2x^2)$.

(a) [5%] Find the force acted on the particle.

(b) [5%] Write down the Newton equation of motion for the particle.

(c) [15%] Show that the time required for the particle to reach the force center from a distance d is d^2/k .

3. A system is composed of n particles, with each particle's mass described by m_a , where $a=1, 2, \dots, n$. The total mass of the system is denoted by M . Show that

(a) [5%] The linear momentum of the system (\mathbf{P}) is the same as if a single particle of mass M were located at the position of the center of mass and moving in the manner the center of mass moves, i.e., $\mathbf{P} = M\mathbf{V}$, \mathbf{V} is the velocity of the center of mass.

(b) [15%] The time differential of the linear momentum of the system is equal to the sum of all the external forces (\mathbf{F}_e), i.e. $d\mathbf{P}/dt = \mathbf{F}_e$, as long as the internal forces follow $\mathbf{f}_{ab} = -\mathbf{f}_{ba}$, the weak form of Newton's Third Law, where \mathbf{f}_{ab} is the force acted on the particle a due to the particle b .

(c) [5%] The total linear momentum for a system free of external forces is constant and equal to the linear momentum of the center of mass (the law of conservation of linear momentum for a system).

4. A particle of mass m moves in a plane under a central forces field with the potential energy $U(r) = \frac{1}{2} k r^2$, where k is a constant, $r^2 = x^2 + y^2$, (x, y) are rectangular coordinates of the particle.

(a) [2%] Write down the Lagrangian of the system.

(b) [5%] Write down the Lagrangian equations for coordinates x and y , respectively.

(c) [15%] At $t=0$, the particle is at rest and at position (A, B) , solve the Lagrangian equations to find $x(t)$ and $y(t)$ at any time t .

(d) [3%] At what condition will the particle move along a circle?