

淡江大學 99 學年度轉學生招生考試試題

系別：物理學系三年級

科目：應 用 數 學

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Please do your best to solve the following problems.

1. Show that $f(x) = \frac{1}{\pi} \frac{\varepsilon}{x^2 + \varepsilon^2}$ approaches to $\delta(x)$ as $\varepsilon \rightarrow 0^+$. (10 points)
2. Write down the definition of Levi-Civita symbol ε_{ijk} and show that $\sum_{i,j,k=1}^3 \varepsilon_{ijk} = 0$. (10 points)
3. Evaluate the area enclosed by a simply connected curve $C : x^{2/3} + y^{2/3} = 4$ in the x - y plane. (15 points)
4. Show that the sum of the square of the matrix elements is invariant under orthogonal similarity transformation. (15 points)
5. Given $A = \begin{pmatrix} -3 & 2 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, and $f(A) = A^3 - 2A^2 + 5A - 3$, find $f(A)$. (20 points)

Note: Evaluating each term in $f(A)$ separately and then adding it up are not credited.

6. Solve the following 2nd-order ordinary differential equation. (15 points)

$$y'' + 2y' + y = 4e^{-x}.$$

7. Evaluate $I = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx$. (15 points)