淡江大學98學年度轉學生招生考試試題

系別：物理學系三年級　　　　　科目：應　用　數　學

准帶項目請打「V」	
	計算機

本試題共 6 大題，/ 頁

1. Given $f_n = \int_0^\infty x^n \exp(-x^2)dx$, show that (a) $f_{2n+1} = n!/2$ and (b) $f_{2n} = 2^{-n}(2n-1)(2n-3).....(5)(3)(1)f_0$. (15 points)

2. The function $f(x,y)$ satisfies the following differential equation $y\dfrac{\partial f}{\partial x} + x\dfrac{\partial f}{\partial y} = 0$. By changing to new variables $u = x^2 - y^2$ and $v = 2xy$, show that f is a function of $x^2 - y^2$ only. (15 points)

3. The commutator $[X,Y]$ of two matrices is defined by $[X,Y] \equiv XY - YX$. Two anticommuting matrices A and B satisfy $A^2 = 1$, $B^2 = 1$, and $[A,B] = 2iC$. (a) Prove that $C^2 = 1$ and $[B,C] = 2iA$. (b) Evaluate $[[[A,B], [B,C]], [A,B]]$. (20 points)

4. Evaluate $I = \oint_c [y(4x^2 + y^2)dx + x(2x^2 + 3y^2)dy]$ around the ellipse $(x^2/a^2) + (y^2/b^2) = 1$. (20 points)

5. Find the Fourier series of the function $f(x) = x$ in the range $-\pi < x \le \pi$ and evaluate the infinite series $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} - \dfrac{1}{11} + ...$ (15 points)

6. Find the solution of the following differential equation $(1-x^2)y' + 2xy = (1-x^2)^{3/2}$. (15 points)