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淡江大學 97 學年度轉學生招生考試試題

系別：物理學系三年級

科目：應 用 數 學

可否使用計算機			
可	否	否	✓

本試題共 5 大題， 1 頁

※ 每大題 20 分！

1. (a) $\Phi(x, h) = (1 - 2xh + h^2)^{-1/2} \equiv \sum_{\ell=0}^{\infty} P_{\ell}(x)h^{\ell}$, where $|h| < 1$;

Find the functions $P_0(x) = ?$, $P_1(x) = ?$, $P_2(x) = ?$ and $P_3(x) = ?$

(b) $\Phi(x, h) = e^{(1/2)x(h-h^{-1})} \equiv \sum_{n=-\infty}^{\infty} J_n(x)h^n$;

Find the functions $J_0(x) = ?$, $J_1(x) = ?$, $J_{-1}(x) = ?$ and $J_2(x) = ?$

2. Find the eigenvalues and corresponding eigenvectors of the following matrices.

(a) $A = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$ (b) $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

3. Find the solution of the following differential equations.

(a) $y'' + y' - 2y = e^x$

(b) $y'' - 2y' + y = 4\sin 2x$

4. (a) Expand the periodic function in a sine-cosine Fourier series.

$$F(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

(b) Qualitatively sketch a graph of the sum of three terms of the series.

[Hint: Sketch each term separately on the same axes and add the terms graphically.]

5. (a) Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx = ?$

(b) Evaluate $\int_{-\infty}^{\infty} \left[\frac{a}{x^2 + a^2} \right] e^{-i\alpha x} dx = ?$, for $\alpha > 0$.