淡江大學96學年度轉學生招生考試試題

系別：物理學系三年級　　科目：理 論 力 學

1. The unit vectors in spherical coordinate system are $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$. In terms of the unit vectors in Cartesian coordinate system, $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$, we have $\mathbf{e}_r=\sin\theta\cos\phi\,\mathbf{e}_x+\sin\theta\sin\phi\,\mathbf{e}_y+\cos\theta\,\mathbf{e}_z$ and $\mathbf{e}_\theta=\cos\theta\cos\phi\,\mathbf{e}_x+\cos\theta\sin\phi\,\mathbf{e}_y-\sin\theta\,\mathbf{e}_z$.
(a) Find expression for $\mathbf{e}_\phi$ in terms of $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$. [3%]
(b) Find expressions for velocity vector v and acceleration vector a in spherical coordinates. [12%]

2 Consider the one-dimensional potential

$$U(x)=\frac{w}{d^4}\left(x^4-4d^2x^2-d^4\right)e^{-(x/d)^2},$$

where w and d are positive constants.
(a) Find equilibrium points of the potential and justify which is (are) the stable equilibrium point(s). [10%]
(b) Sketch the potential. [3%]
(c) A particle of mass m moves in the force field given above, find the force acting on this particle. [5%]
(d) Is the momentum of the particle conserved? Why? [2%]
(e) A particle with energy E=-0.5w moves in the force field given above, is the motion bounded? How many turning points for the motion if it is bounded? [7%]
(f) Another particle with energy E=0.5w moves in the same field, is the motion bounded? Why? [3%]

Data may be useful for problem 2: $6^{1/2}\approx 2.449$, $\exp(6^{1/2}-3)\approx 0.5767$, $\exp(-6^{1/2}-3)\approx 0.0043$.

3. A one-dimensional simple harmonic oscillator consists of a particle of mass m attached to a massless Hooke spring whose force constant is k. At t=0, the particle is displaced a distance x_0 from the equilibrium position (x=0) and released from rest, and a driving force $F(t)=F_0\sin(\omega t)$ is applied to the oscillator (F_0 and ω are constants).
(a) Find $x(t)$ at t>0. [15%]
(b) Find $x(t)$ by taking the limit $\omega\rightarrow\omega_0=(k/m)^{1/2}$ in your result for part (a). What will occur when $\omega\rightarrow\omega_0$? [5%]

4. Consider a vertical plane in a constant gravitational field. Let the origin of a coordinate system be located at some point in this plane. A particle of mass m moves in the vertical plane under the influence of gravity and an additional force $f=-A\,r^{\alpha-1}$ directed toward the origin (r is the distance from the origin; A and α[$\neq$0 or 1] are constants).
(a) Choose appropriate generalized coordinates and find Lagrangian equations of motion [15%].
(b) Is the angular momentum about the origin conserved? Explain. [5%]

5 A projectile is fired at an angle 45° with initial kinetic energy E_0. At the top of its trajectory, the projectile explodes with additional energy E_0 into two fragments. One fragment of mass m_1 travels straight down.
(1) What is the velocity (magnitude and direction) of the second fragment of mass m_2? [7%]
(2) What is the velocity of the first fragment of mass m_1? [4%]
(3) What is the ratio of m_1/m_2 when m_1 is a maximum? [4%]