淡江大學 95 學年度轉學生招生考試試題

系別：物理學系三年級　　　　科目：理 論 力 學

35-1

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

1. If $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$ denote respectively the position, velocity, and acceleration of a particle, prove that

$$\frac{d}{dt}\left[\mathbf{a}\cdot(\mathbf{v}\times\mathbf{r})\right] = \dot{\mathbf{a}}\cdot(\mathbf{v}\times\mathbf{r}). \quad (10\%)$$

2. Find expressions for the unit vectors $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ of spherical polar coordinates in terms of the Cartesian $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$. Differentiate these expressions with respect to time to find $d\hat{\mathbf{r}}/dt$, $d\hat{\boldsymbol{\theta}}/dt$, and $d\hat{\boldsymbol{\phi}}/dt$. (The components can be expressed as functions of r, θ, ϕ, $\dot{r}$, $\dot{\theta}$, and $\dot{\phi}$.) (18%)

3. Consider a particle of mass m conducting a one-dimensional motion. The speed of the particle varies with the distance x as $v(x) = Ax^{-2}$. Assume $x(t=0) = 0$.

   (a) Find the force $F(x)$ responsible for this motion. (10%)

   (b) Determine $x(t)$. (10%)

   (c) Derive $F(t)$. (4%)

4. A mass $m = 1$ kg is tied to one end of a spring with force constant 1 N/m and with the other end held fixed. The initial displacement (with respect to the place where no force is exerted on the mass by the spring) and velocity for the mass are $x(t=0) = 0$ and $v(t=0) = 1$ m/s.

   (a) What are the angular frequency ω, the frequency f, and the period τ of its oscillations? (6%)

   (b) The general solution of this one-dimensional simple harmonic motion can be represented as

$$\begin{aligned} x(t) &= A_1 e^{i\omega t} + A_2 e^{-i\omega t} \\ &= B_1 \cos\omega t + B_2 \sin\omega t \\ &= C\cos(\omega t - \delta) \\ &= \mathrm{Re}\, D e^{i\omega t}. \end{aligned}$$

   Determine the values of A_1, A_2, B_1, B_2, C, D, and δ. (14%)

   (c) Derive the total energy of this system and the maximum displacement of the mass. (4%+2%)

5. Two particles of equal mass, $m_1 = m_2 = m$, are confined to the x axis and connected by a spring with potential energy $U = \frac{k}{2}x^2$. (Here x is the extension of the spring, $x = x_2 - x_1 - l$, where l is the spring's unstreched length and particle 1 is assumed to always remain to the left of particle 2.)

   (a) Write down the Lagrangian $\mathcal{L}(x_1, x_2, \dot{x}_1, \dot{x}_2)$ and the corresponding Lagrange's equations of motion. (4%+3%+3%)

   (b) Rewrite $\mathcal{L}$ in terms of the new variables x (the extension of the spring) and $X = \frac{1}{2}(x + x_2)$ (the position of the center of mass of these two particles), and write down the corresponding Lagrange's equations of motion. (6%+3%+3%)