淡江大學 95 學年度轉學生招生考試試題

系別：物理學系三年級　　科目：應 用 數 學　　37-1

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

1. (a) Two Hermitian matrices A and B have the same eigenvalues. Show that A and B are related by a unitary similarity transformation. (10 points)

   (b) Show that the sum of the square of the matrix elements is invariant under orthogonal similarity transformation. (10 points)

2. Write down the mathematical expression of convolution associated with two given functions $f(t)$ and $g(t)$ and briefly explain how convolution can be employed in spectroscopy. (20 points)

3. Evaluate $I = \iint_s \vec{a} \cdot d\vec{\sigma}$, where $\vec{a} = x\hat{i}$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \geq 0$. (20 point)

4. Show that the Fourier series for the function $y(x) = |x|$ in the range of $-\pi \leq x < \pi$ is $y(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^2}$ and deduce the sum of the infinite series $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + ... = ?$ (20 points)

5. Evaluate $I = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx$. (20 points)