淡江大學九十四學年度轉學生招生考試試題

系別： 物理學系三年級　　　科目：理　論　力　學

准帶項目請打「V」	
X	簡單型計算機

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1. A stone is thrown to the sky from the origin (so that $\mathbf{r}(t=0)=0$) with the initial velocity $\mathbf{v}_0=\hat{\mathbf{i}}v_0\cos\theta+\hat{\mathbf{k}}v_0\sin\theta$, where $0\le\theta\le\pi/2$ is the elevation angle. The gravational acceleration is given by $\mathbf{g}=-g\hat{\mathbf{k}}$. ($\hat{\mathbf{i}}$ and $\hat{\mathbf{k}}$ are the unit vectors along the x- and z-directions, respectively.)
   - (a) Derive the position vector $\mathbf{r}(t)$ for the stone in terms of $\mathbf{v}_0$ and $\mathbf{g}$? [4%]
   - (b) To ensure that the stone is always moving away from the thrower, there must be $0\le\phi<\phi_0$. Find the value of $\sin\phi_0$. [18%]

2. A simple harmonic oscillator consists of a 1-kg mass attached to a spring with force constant 1 nt/cm. The mass is displaced 10 cm and released from rest (i.e. $x(t=0)=0.1$ m where x denotes the displacement with respect to the equilibrium place for the spring).
   - (a) The general solution of this one-dimensional simple harmonic motion can be represented as

$$\begin{aligned} x(t) &= A_1e^{i\omega t}+A_2e^{-i\omega t} \\ &= B_1\cos\omega t+B_2\sin\omega t \\ &= C\cos(\omega t-\delta) \\ &= \mathrm{Re}\,De^{i\omega t}. \end{aligned}$$

     Determine the values of A_1, A_2, B_1, B_2, C, D, ω, and δ. (Don't forget the units.) [2%×6+3%×2]
   - (b) Derive the total energy of this system and the maximum speed of the mass. [10%]

3. The Lagrangian for a single particle in two dimensions is given by

$$\mathcal{L}=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-U(x,y),$$

   where U is the potential energy.
   - (a) Write down the two Lagrange's equations for the coordinates x and y, respectively. [6%]
   - (b) Express the Lagrangian in polar coordinates (ρ,ϕ). [6%]
   - (c) Write down the two Lagrange's equations for the coordinates ρ and ϕ, respectively. [10%]

4. Consider a rigid body composed of n particles of masses m_α ($\alpha=1,2,\ldots,n$), positioned at $\mathbf{r}_\alpha$ with respect to the center of mass of this rigid body. The motion of this body can be decomposed into two parts: a translation (of the center of mass) with the instantaneous linear velocity $\mathbf{V}$ relative to the ground and a rotation with the instantaneous angular velocity ω with respect to the center of mass.
   - (a) What is the velocity $\mathbf{v}_\alpha$ of the particle α relative to the center of mass (expressed in terms of $\mathbf{r}_\alpha$, ω, and $\mathbf{V}$). [6%]
   - (b) Express the body's angular momentum $\mathbf{L}=\sum_\alpha m_\alpha\mathbf{r}_\alpha\times\mathbf{v}_\alpha$ in terms of m_α, $\mathbf{r}_\alpha$, ω, and $\mathbf{V}$). [4%]
   - (c) Express the Cartesian components of the angular momentum as $L_i=\sum_j I_{ij}\omega_j$, where the subscripts i and j designating the components in the Cartesian coordinates. Write down the 9 elements I_{ij} of the inertia tensor $\{\mathbf{I}\}$ in terms of m_α and the components $(x_\alpha,y_\alpha,z_\alpha)$ of $\mathbf{r}_\alpha$. [18%]