淡江大學九十三學年度轉學生招生考試試題 37-1

系別：物理學系三年級　　科目：理 論 力 學

准帶項目請打「○」否則打「×」	
X	簡單型計算機

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1. Consider a particle of mass m constrained to move on the surface of a sphere of radius R subject to an applied force $\mathbf{F}(\theta,\phi) = F_\theta(\theta,\phi)\,\mathbf{e}_\theta + F_\phi(\theta,\phi)\,\mathbf{e}_\phi$. Here, we use the spherical coordinate systems (r,θ,ϕ) with the center of the sphere as the origin. Write down the equations of motion along the θ-direction and the ϕ-direction, respectively. [20%]

2. The equation of motion of a damped oscillator can be expressed as

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0.$$

Given the initial condition at $t = 0$ by $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$, determine the explicit solutions of $x(t)$ in the cases of underdamping ($\omega_0^2 > \gamma^2$), critical damping ($\omega_0^2 = \gamma^2$), and overdamping ($\omega_0^2 < \gamma^2$), respectively. (30%)

3. Consider a rigid body composed of n particles of masses m_α ($\alpha = 1, 2, \ldots, n$), positioned at $\mathbf{r}_\alpha$ with respect to the center of mass of this rigid body. The motion of this body can be decomposed into two parts: a translation (of the center of mass) with the instantaneous linear velocity $\mathbf{V}$ relative to the ground and a rotation with the instantaneous angular velocity ω with respect to the center of mass. What is the velocity $\mathbf{v}_\alpha$ of the particle α relative to the ground (expressed in terms of $\mathbf{r}_\alpha$, ω, and $\mathbf{V}$). [6%] Show that the total kinetic energy T of this rigid body can also be divided into two parts: a translational part T_{trans} (independent of ω) and a rotational part T_{rot} (independent of $\mathbf{V}$). [10%] The rotational part can be expressed as $T_{\text{rot}} = \frac{1}{2}\sum_{i,j=1}^{3} I_{i,j}\omega_i\omega_j$, where the subscripts i and j designating the components in the rectangular coordinates. Write down the 9 elements I_{ij} of the inertia tensor $\{\mathbf{I}\}$ in terms of m_α and components of $\mathbf{r}_\alpha$. [10%]

4. Consider the double pulley system shown below. Assume the pulleys are massless and both strings across the pulleys are of fixed length $l + \pi R$, where R is the radius of the pulleys. Use **the method of Lagrange undetermined multiplier** to find the tensions in both strings. [24%]