淡江大學九十二學年度轉學生招生考試試題

系別：物理學系三年級　　科目：電　磁　學

准帶項目請打「○」否則打「×」	
×	簡單型計算機

1. Consider an electrostatic situation in which a charge Q is uniformly distributed over a sphere of radius R. Let the electric potential at infinity be zero.
   (a) Find the electric field ($\vec{E}$) and potential (V) as a function of the radial distance (r) from the center of the sphere.
   (b) Calculate the electrostatic energy of the system. (20%)

2. Assume that a static charge distribution (ρ) produces an electric potential $V(\vec{r}) = A\dfrac{e^{-\alpha r}}{r}$, where A and α are positive constants, $\vec{r}$ is the position vector, $r = |\vec{r}|$. Now, it is given that $\nabla^2(1/r) = -4\pi\delta(\vec{r})$. Find the electric field $\vec{E}(\vec{r})$, charge density $\rho(\vec{r})$, and the total charge Q. (20%)

3. Jusitify that the vector potential of a uniform magnetic field $\vec{B}$ may be expressed as $\vec{A} = \vec{B} \times \vec{r}/2$, where $\vec{r}$ is the position vector, and that $\nabla \cdot \vec{A} = 0$. (10 %)

4. Assume that a long solenoid of radius R with n turns of thin wires per unit length is carrying a steady current I, and its axis of symmetry is along the z-direction.
   (a) Justify that the magnetic field (whenever nonzero) is in the z-direction.
   (b)Show that $\oint_C \vec{A} \cdot d\vec{l} = \Phi_B$ (the magnetic flux through a surface enclosed by a closed path C), where $\vec{A}$ is the vector potential.
   (c) Find the magnetic field intensity ($\vec{H}$) and vector potential throughout the space. (20%)

5. It is well known that the basic laws of classical electromagnetic phenomena are governed by the Maxwell's equations: $\nabla \cdot \vec{D} = \rho_f$, $\nabla \times \vec{H} = \vec{J}_f + \dfrac{\partial \vec{D}}{\partial t}$, $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$, $\nabla \cdot \vec{B} = 0$, where $\vec{E}$, $\vec{D}$, $\vec{B}$, $\vec{H}$, ρ_f, and $\vec{J}_f$ are the electric field, electric displacement, magnetic field, magnetic field intensity, free charge density, and free current density, respectively.
   (a) Write down the corresponding integral form for each of the four Maxwell's equations, and specify the physical meaning for each of them.
   (b) Show that the free charge density and free current density satisfy the continuity quation: $\nabla \cdot \vec{J}_f + \dfrac{\partial \rho_f}{\partial t} = 0$ (Hint: Two of the Maxwell equations may be useful).
   (c) Show that the tangential component of the electric field must be continuous across any boundary surface. (30 %)