淡江大學八十八學年度日間部轉學生招生考試試題

系別：物理系三年級　　科目：應用數學

本試題共 / 頁

共五大題，每題各佔20分.

1. Find the general solutions $y(t)$ and $z(t)$ for the coupled differential equations:

$$a\frac{dz}{dt} = m\frac{d^2y}{dt^2} \quad \text{and} \quad b - a\frac{dy}{dt} = m\frac{d^2z}{dt^2}$$

where

a, b, and m are constants.

2. Solve the differential equation:

$y'' + k^2y = \cos mx$ where m and k are constants, and

$m \neq 0$, $k^2 \neq m^2$.

3. Show that $(\vec{A} \times \vec{B}) \times \vec{C}$ equals to one and only one of the followings: ⓐ $\vec{A}(\vec{B}\cdot\vec{C}) - \vec{C}(\vec{B}\cdot\vec{A})$

ⓑ $\vec{B}(\vec{C}\cdot\vec{A}) - \vec{A}(\vec{C}\cdot\vec{B})$

ⓒ $\vec{C}(\vec{A}\cdot\vec{B}) - \vec{B}(\vec{A}\cdot\vec{C})$,

for any vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.

4. Solve the differential equation:

$$(x^2 + by^2)\frac{dy}{dx} + 3x^2 + 2xy = 0.$$

5. Evaluate the integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + \cos^2 x}\, dx.$$