淡江大學 103 學年度日間部轉學生招生考試試題 64

系別：統計學系三年級　　科目：機率與微積分

考試日期：7 月 20 日(星期日) 第 3 節　　本試題共 6 大題， 1 頁

1. (20 points) Let the probability density function of a random variable X is $f(x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0,1,\ldots,n$. Prove the expectation and variance of X are $E(X) = np$ and $Var(X) = np(1-p)$, respectively

2. (15 points) Let random variables X and Y are independent and follow N(0,1). Prove that $M_{X+Y}(t) = M_X(t)M_Y(t)$, where $M_Z(t)$ denotes the moment generating function of the random variable Z.

3. (15 points) The joint density function of random variables X and Y is given by

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y}, 0 < x < \infty, 0 < y < \infty, \\ 0, otherwise \end{cases}$$

Compute the probability P(X<Y).

4. (15 points) Let E and F be two events in the sample space S with the probability P(E)=0.7 and P(F)=0.5, respectively. Assume that $P(E \cup F) = 0.8$. Find the probability $P(E \cap F)$.

5. (20 points) If the random variable X has an exponential distribution with the probability density function

$$f(x) = \lambda e^{-\lambda x}, x > 0.$$

Prove that $P(X > s + t | X > t) = P(X > s)$ for all s,t >0.

6. (15 points) Find the value of $\int_1^4 x^2 dx$.