淡江大學 102 學年度日間部轉學生招生考試試題

系別：統計學系三年級　　　　　　　　科目：機率與微積分

考試日期：7月24日(星期三) 第3節　　　　本試題共　7　大題，　　1　頁

1. Calculate $\int_0^\infty \int_0^{y/2} \lambda^2 e^{-\lambda x - \lambda y}\, dxdy$. (10%)

2. Let the independent random variables X and Y have the Exponential distribution with parameter $\lambda = 1$. Set $U = X + Y$ and $V = X/Y$.

    (1) Derive the joint p.d.f. of U and V . (20%)

    (2) Are the random variables U and V independent? Why? (8%)

3. If for three events A, B, C it happens that either $A \cup B \cup C = A$ or $A \cap B \cap C = A$, what conclusions can you draw? (8%)

4. Let $Z \sim N(0,1)$, derive the moment generating function $M_Z(t)$. (12%)

5. Let the joint p.d.f. of X and Y be $f_{X,Y}(x,y) = \dfrac{2}{n(n+1)}$ for $y = 1, \cdots, x$ and $x = 1, \cdots, n$, where n is a given constant.

    (1) Determine the conditional probability density function $f_{Y|X}(\cdot \mid x)$. (12%)

    (2) Calculate $E(Y \mid X = x)$. (10%)

6. Evaluate the integral $\int_{-\infty}^{\infty} \dfrac{1}{1+x^2}\, dx$. (10%)

7. Find the value of $\partial x / \partial z$ at the point $(1,-1,-3)$ if the equation $xz + y \ln x - x^2 + 4 = 0$ defines x as a function of two independent variables y and z and the partial derivative exists. (10%)