

# 淡江大學 100 學年度轉學生招生考試試題

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系別：統計學系三年級

科目：機率與微積分

考試日期：7 月 19 日(星期二) 第 3 節

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1.

(1) Please explain the relationship between "Probability Theory" and "Statistical Inferences"(e.g. estimation, tests, and confidence intervals) .

(2) Please explain the relationship among the concepts of limits, continuity, differentiation, and integration. You can use examples for demonstration in your explanation.

2. If the random variable  $X$  is the outcome of throwing a fair dice, then the sample space of  $X$  is  $\{1, 2, 3, 4, 5, 6\}$ . Please use **Riemann-Stieltjes integral** to calculate the expectation and variance of  $X$ , that is,  $E[X]$  and  $\text{Var}[X]$ .

3. Please find the density function of the new random variable  $W = |X|$ , where  $X$  is a random variable following the  $t$  distribution with  $n$  degrees of freedom, and has the following density function :

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi} \Gamma\left(\frac{n}{2}\right)} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}, -\infty < x < \infty$$

Hint:  $\Gamma(m)$  is the gamma function  $\int_0^{\infty} e^{-x} x^{m-1} dx$ ,  $m > 0$ .

4. Discuss the continuity of the following function, and indicate the points of discontinuity(if any):

$$f(x) = \begin{cases} \left[\frac{x-1}{2-x}\right]^{1/2}, & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

5. Find the Taylor's expansion of the function  $f(x) = \log(1+x)$  at  $x=0$ . Please show at least the first four terms of the Taylor's expansion.