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淡江大學 99 學年度轉學生招生考試試題

系別：統計學系三年級　　　　科目：統　計　學

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1) (a) What is meant by the n random variables $X_1, X_2, \ldots, X_n$ is a random sample (of size n)? (4%)

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n. Using it, define the following: (18%)

(b) Sample mean.

(c) Sample median.

(d) Sample mode.

(e) Sample variance.

(f) Sample standard deviation.

(g) Sample $100p\%$ .percentile $(0 < p < 1)$.

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2) Let Z be a standard normal random variable.

(a) What is the probability density function of Z? Draw a picture for it. Is the picture symmetric about the line $z = 0$? Why? (6%)

(b) Let F_Z be the distribution function of Z. That is, $F_Z(z) = P(Z \le z)$, for any real number z. Express the probability $P(-a < Z \le a)$ in terms of F_Z. (3%)

(c) If $F_Z(-a) = 0.123$, find $1 - F_Z(a)$. (3%)

(d) If $F_Z(a) = 0.876$, find $F_Z(-a)$ (3%)

3) The probability density functions of the independent random variables X and Y are given by

$$f_X(x) = \begin{cases} 0.2, & if\ x = 0 \\ 0.8, & if\ x = 1 \\ 0, & otherwise, \end{cases} \qquad f_Y(y) = \begin{cases} 0.2, & if\ y = 1 \\ 0.4, & if\ y = 2,3 \\ 0, & otherwise \end{cases}$$

(a) Find the joint probability density function of X and Y. (6%)

(b) Find $Cov(U,V)$, the covariance of $U = 2X + 1$ and $V = Y + 2$. (4%)

4) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a population with mean μ and variance $\sigma^2 > 0$.

(a) Give an unbiased estimator of μ. (3%)

(b) Give an unbiased estimator of σ^2, if the population mean μ is unknown. (4%)

(c) Suppose that the population is a normal population with known population variance σ^2, Find a 95% confidence interval estimator of the population mean μ. (6%)

(For reference, $z_{0.1} = 1.281$, $z_{0.05} = 1.645$, $z_{0.025} = 1.960$, , $z_{0.005} = 2.576$)

背面尚有試題

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5) In testing the null hypothesis H_0 against the alternative hypothesis H_1 by using a test. (15%)

(a) What is meant by the critical region for the test?

(b) What is meant by the level of significance of the test?

(c) When a type I error is committed?

(d) When a type II error is committed?

(e) What is the power of this test?

6) In a simple linear regression model $Y = \alpha + \beta x + \varepsilon$, the responses Y_i corresponding to the input values x_i, $i = 1,2,\ldots,n$, are observed and used to estimate the parameters α and β. Assume that the responses variables Y_i, $i = 1,2,\ldots,n$, are independent.

(a) What are the least-square estimators $\hat{\alpha}$ and $\hat{\beta}$ of α and β, respectively?. (8%)

(b) What is the estimated regression line? (3%)

(c) Let the error random variable ε be normal distributed with mean 0 and variance σ^2.

(i) What is the distribution of the response variable Y? (4%)

(ii) What are the distributions of $\frac{\sum_{i=1}^{n}(Y_i - \alpha - \beta x_i)^2}{\sigma^2}$ and $\frac{\sum_{i=1}^{n}(Y_i - \hat{\alpha} - \hat{\beta} x_i)^2}{\sigma^2}$, respectively? (6%)

(iii) Find the unbiased estimator of σ^2. (4%)