淡江大學99學年度轉學生招生考試試題

系別：統計學系三年級　　　　　科目：機率與微積分

本試題共　8　大題，　　1 頁

1. Evaluate the following limits:

   (a) $\lim\limits_{x \to \infty}\left(x + 1 - \sqrt{x^2 + x + 1}\right)$　　(5%)

   (b) $\lim\limits_{x \to 0}\dfrac{1}{x^3}\displaystyle\int_0^{x^2} \sin\sqrt{t}\,dt$　　　(5%)

2. Evaluate the following integral:

   (a) $\displaystyle\int x \tan^{-1} x\,dx$　　　　(8%)

   (b) $\displaystyle\int \dfrac{\sqrt{1 - x^2}}{x}\,dx$　　　(8%)

   (c) $\displaystyle\int_0^1 \int_0^x \sqrt{x^2 + y^2}\,dy\,dx$　　(8%)

3. Let $f(x) = |x + 2|$, prove $f'(-2)$ does not exist.　(8%)

4. Find the shortest distance from the point $(1, 0)$ to the parabola $y^2 = 4x$.　(8%)

5. The probability function of X is given by
   $f(x) = (1 - p)^{x-1}p, x = 1, 2, 3, \cdots, 0 < p < 1$. Find
   (a) $E(X)$　　　(4%)
   (b) $Var(X)$　　　(4%)

6. Let $E(x^k) = 2^k, k = 1, 2, 3, \cdots$, find the moment-generating function of X.　(8%)

7. The joint probability distribution of X and Y is given by
   $f(x, y) = \begin{cases} 3y, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$
   (a) Find $Cov(X, Y)$　　(8%)
   (b) Find $E(Y \mid X = x)$　　(8%)
   (c) Find the probability distribution of $Z = Y - X$　　(10%)

8. Let X and Y be independent with normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$,

   respectively. Prove the probability distribution of $X - Y$ is $N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$.　(8%)