淡江大學 98 學年度轉學生招生考試試題

系別：統計學系三年級　　　　　科目：機率與微積分

1. If the events D, E and F are related as follows:

   $$D \subset E \subset F \quad \text{and} \quad P(D) = \frac{1}{4}, \quad P(E) = \frac{5}{12}, \quad P(F) = \frac{7}{12},$$

   Evaluate the following probabilities :

   (1)　$P(D^C \cap E)$　　　　　　　　　　　(6%)

   (2)　$P(D^C \cap E^C \cap F^c)$　　　　　　　(6%)

2. Let the random variables X, Y be jointly distributed with p.d.f. given by

   $$f(x,y) = \begin{cases} e^{-x-y} & 0 < x < \infty, \ 0 < y < \infty \\ 0 & otherwise. \end{cases}$$

   (1)　Are X and Y independent ? Why ?　　　　　　(6%)

   (2)　Let W=X+Y. Find the moment generating function of W.　　(6%)

3. Let the random variable X have the Poisson distribution with parameter λ . Calculate

   (1) E(X)　　　　　　　　　　(8%)

   (2) V(X)　　　　　　　　　　(8%)

4. Suppose that the random variable Z is distributed as N(0,1). Find the probability density function of Z^2 .　　　　　　(10%)

5. Discuss the continuity of the function below at 0.

   $$f(x) = \begin{cases} x^2 + 1 & if \ x \neq 0 \\ 2 & if \ x = 0 \end{cases} \qquad (10\%)$$

6. Find the following derivative or integral:

   (1)　$f'(x), where \ f(x) = x(x^2 + 1)^3.$　　　　(8%)

   (2)　$f'(x), where \ f(x) = x^x.$　　　　　　(8%)

   (3)　$\int_0^2 x\sqrt{4 - x^2} \ dx.$　　　　　　　(8%)

7. Find the area enclosed by the graphs of $y = x^3$ and $y = x$.　　(8%)

8. Write the Taylor polynomial of degree n for $f(x) = e^x$ at 0.　　(8%)