淡江大學97學年度轉學生招生考試試題

系別：統計學系三年級　　　　科目：機率與微積分

可否使用計算機			
可		否	✓

本試題共 8 大題， 2 頁

（請附計算和推導過程，否則不予計分，背面仍有試題）

1. Evaluate the following limits:

(a) $\lim_{h\to 0}\frac{(1+h)^{10}-1}{h}$ (6%)　(b) $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n-1}{n^2}+\frac{n}{n^2}\right)$ (10%)

2. Evaluate the following integral (indefinite and definite)

(a) $\int\frac{4x^3}{1-x^4}\,dx$ (6%)　(b) $\int_0^{\pi}x\sin x\,dx$ (7%)

3. Let

$$f(x)=\begin{cases}\dfrac{4x-2\sin 2x}{4x^3} & x\neq 0\\ \alpha & x=0\end{cases}$$ (10%)

Find the value of α such that the function f is everywhere continuous

4. Assume that two variables X and Y are randomly chosen from $Uniform(-1,1)$ and $Uniform(0,1)$, respectively. (i.e. $X\sim Uniform(-1,1)$ and $Y\sim Uniform(0,1)$ are independent). Find the probability that the roots of the equation $h(t)=0$ are real, where $h(t)=t^2+2Xt+Y$ （亦即 h 之根為實數的機率為何？） (8%)

5. Let X be a **positive** continuous random variable with probability density function (p.d.f.) f and cumulative distribution function $F(x)=\int_0^x f(t)dt$. Let h be defined as (10%)

$$h(x)=\frac{f(x)}{1-F(x)}=\lambda,\ \forall x>0,$$

where λ is a positive constant. Prove that the p.d.f. f has the form $f(x)=\lambda e^{-\lambda x}, x>0$. (Hint: integrate h with respective to x and note that $F'=f$).

6. Let Y be a continuous random variable with p.d.f. $f_Y(y)=ky^2+6y, y\in[0,1]$. Calculate (a) the value of k (b) the median of Y (c) the mode of Y (5%, 5%, 5%)

本試題雙面印製

◀注意背面尚有試題▶

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7. Suppose that A and B are events such that $P(A) = 1/3$, $P(B) = 1/5$ and $P(A \mid B) + P(B \mid A) = 2/3$. Evaluate $P(A^c \cap B^c)$ (8%)

8. Suppose that the joint p.d.f. of two random variables X and Y

$$f(x,y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}, \quad -\infty < x < \infty, -\infty < y < \infty .$$

(a) Are X and Y independent? Briefly Explain. (5%)

(b) Find $P(X^2 < 1/4 \text{ and } Y^2 > 1)$. (8%)

(c) Derive the p.d.f of $W = X^2$ (9%)

($\Phi(1/2) = 0.6915$, $\Phi(1) = 0.8413$, $\Phi(2) = 0.9772$, where $\Phi(x) = \int_{-\infty}^{x} e^{-t^2/2} / \sqrt{2\pi}\, dt$)