## 本試題雙面印製

## 淡江大學 97 學年度轉學生招生考試試題

系別:統計學系三年級

科目:機率與微積分

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(請附計算和推導過程,否則不予計分,背面仍有試題)

- 1. Evaluate the following limits:
  - (a)  $\lim_{h\to 0} \frac{(1+h)^{10}-1}{t_1}$ (6%)
- (b)  $\lim_{n\to\infty} \left( \frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2} + \frac{n}{n^2} \right)$  (10%)
- 2. Evaluate the following integral (indefinite and definite)
  - (a)  $\int \frac{4x^3}{1-x^4} dx$
- (6%) (b)  $\int_0^{\pi} x \sin x \, dx$  (7%)
- 3. Let

1.1

$$f(x) = \begin{cases} \frac{4x - 2\sin 2x}{4x^3} & x \neq 0\\ \alpha & x = 0 \end{cases}$$
 (10%)

Find the value of  $\alpha$  such that the function f is everywhere continuous

- 4. Assume that two variables X and Y are randomly chosen from Uniform(-1,1)and Uniform(0,1), respectively. (i.e.  $X \sim Uniform(-1,1)$  and  $Y \sim Uniform(0,1)$  are independent). Find the probability that the roots of the equation h(t) = 0 are real, where  $h(t) = t^2 + 2Xt + Y$  (亦即 h 之根為實數的機率為何?) (8%)
- 5. Let X be a **positive** continuous random variable with probability density function (p.d.f.) f and cumulative distribution function  $F(x) = \int_0^x f(t)dt$ . Let h be defined as (10%)

$$h(x) = \frac{f(x)}{1 - F(x)} = \lambda, \ \forall \ x > 0,$$

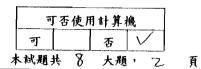
where  $\lambda$  is a positive constant. Prove that the p.d.f. f has the form  $f(x) = \lambda e^{-\lambda x}$ , x > 0. (Hint: integrate h with respective to x and note that F' = f).

6. Let Y be a continuous random variable with p.d.f.  $f_Y(y) = ky^2 + 6y$ ,  $y \in [0,1]$ . Calculate (a) the value of k (b) the median of Y (c) the mode of Y (5%, 5%, 5%)

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- 7. Suppose that A and B are events such that P(A) = 1/3, P(B) = 1/5 and P(A|B) + P(B|A) = 2/3. Evaluate  $P(A^c \cap B^c)$  (8%)
- 8. Suppose that the joint p.d.f. of two random variables X and Y

$$f(x,y) = \frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}, -\infty < x < \infty, -\infty < y < \infty.$$

- (a) Are X and Y independent? Briefly Explain. (5%)
- (b) Find  $P(X^2 < 1/4 \text{ and } Y^2 > 1)$ . (8%)
- (c) Derive the p.d.f of  $W = X^2$  (9%)

 $(\Phi(1/2) = 0.6915, \quad \Phi(1) = 0.8413, \quad \Phi(2) = 0.9772, \text{ where } \Phi(x) = \int_{-\infty}^{x} e^{-t^2/2} / \sqrt{2\pi} \ dt)$