

系別：統計學系三年級

科目：機率與微積分

可否使用計算機			
可		否	✓

本試題共 | 頁

1. Let X have a geometric distribution. Show that

$$P(X > a + b | X > a) = P(X > b), \text{ where } a \text{ and } b \text{ are nonnegative integers. (10\%)}$$

2. Let X have the uniform distribution on $(0,1)$. Find the probability density function for $Y = \tan\left(\pi\left(X - \frac{1}{2}\right)\right)$. (10%)

3. Let X_1, X_2 be a random sample from the probability density function

$$f(x) = e^{-x}, 0 < x < \infty. \text{ Find the probability density function of}$$

$$Y = \frac{X_1}{X_2}. \text{ (10\%)}$$

4. Let X_1, X_2, \dots, X_n be a random sample from $N(\mu, \sigma^2)$,

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1},$$

- (a) Find the mean and variance of S^2 . (10%)

- (b) Show that S^2 converges in probability to σ^2 . (10%)

5. Find the equation for the tangent at the point $(2, -1)$ for

$$x^2 - xy + y^2 = 7. \text{ (8\%)}$$

6. Suppose $f'(x) = \cos^2 2x$, $f(2\pi) = \pi$, find the function $f(x)$. (8%)

7. Evaluate the following derivative or integrals: (24%)

(a) $f'(x)$, where $f(x) = x^{\sin x}$.

(b) $\int x \tan^{-1} x dx$.

(c) $\int_0^1 \int_{2x}^2 e^{y^2} dy dx$.

8. Find the Maclaurin series for the following functions: (10%)

(a) $f(x) = \frac{1}{1+x}$.

(b) $f(x) = \ln(1+x^2)$. (Hint: use (a))