

淡江大學 95 學年度轉學生招生考試試題

系別：統計學系三年級

科目：機率與微積分

72-1

准帶項目請打「V」	
X	簡單型計算機

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本試題雙面印製

Question 1 (15 pts)

Evaluate the following limits:

- (a) (5 pts) $\lim_{x \rightarrow 1} \frac{.7}{5 - 2x}$
- (b) (5 pts) $\lim_{x \rightarrow \infty} x^2 e^{-x}$
- (c) (5 pts) $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$

Question 2 (25 pts)

Evaluate the following derivatives or integrals:

- (a) (5 pts) $\frac{d}{dx} f(x)$, where $f(x) = x \ln x$.
- (b) (5 pts) $\frac{d}{dx} f(x)$, where $f(x) = e^{-2x^3}$.
- (c) (5 pts) $\int \frac{1}{x \ln x} dx$.
- (d) (10 pts) $\int \int_T xy \, dx dy$, where $T = \{(x, y) \mid 0 \leq y \leq x \leq 1\}$.

Question 3 (10 pts)

Find the extreme values of the function $f(x) = 1 + 2x - x^2$, where $x \in [0, 10]$.

Question 4 (10 pts)

Prove each of the following statements. (Assume that any conditioning event has positive probability.)

- (a) (5 pts) If $A \subset B$, then $P(A|B) = P(A)/P(B)$.
- (b) (5 pts) If A and B are disjoint events, then $P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}$.

Question 5 (10 pts)

Let X be a random variable with p.d.f.

x	1	2	3	4	5
$f(x)$	0.1	0.25	0.05	c	0.3

- (a) (3 pts) Find c .
- (b) (3 pts) Obtain $P(X > 2)$.
- (c) (4 pts) Compute $E(X)$.

Question 6 (10 pts)

Let the p.d.f. $f(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere.

- (a) (5 pts) If $f(0) = \frac{1}{4}$, find $E(X^2)$.
- (b) (5 pts) If $f(0) = \frac{1}{4}$ and if $E(X) = \frac{1}{4}$, determine $f(-1)$ and $f(1)$.

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Question 7 (10 pts)

Let X_1 and X_2 be two random variables taking on the values $-1, 0, 1$ with the following respective probabilities:

$$\begin{aligned} f(-1, 1) &= \alpha, & f(-1, 0) &= \beta, & f(-1, -1) &= \alpha \\ f(0, -1) &= \beta, & f(0, 0) &= 0, & f(0, 1) &= \beta & ; \quad \alpha, \beta > 0, \quad \alpha + \beta = \frac{1}{4}. \\ f(1, -1) &= \alpha, & f(1, 0) &= \beta, & f(1, 1) &= \alpha. \end{aligned}$$

Then show that:

- (a) (5 pts) $\rho(X_1, X_2) = 0$;
- (b) (5 pts) X_1 and X_2 are dependent.

Question 8 (10 pts)

Let the random variables X, Y be jointly distributed with p.d.f f given by

$$f(x, y) = \begin{cases} e^{-x-y} & 0 < x < \infty, 0 < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X + Y \leq 3)$.