

淡江大學九十四學年度轉學生招生考試試題

系別：統計學系三年級

科目：機率與微積分

准帶項目請打「V」
<input type="checkbox"/> 簡單型計算機

節次：7月13日第3節

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1) (a) Let A, B be two events. Prove that $P(A \cap B) \geq P(A) + P(B) - 1$ (5%)

(b) Let A, B, E be three events. Prove that $\frac{P(A|E)}{P(B|E)} = \frac{P(A)P(E|A)}{P(B)P(E|B)}$ (6%)

2) Let X be a random variable having p.d.f. given by

$$f_X(x) = \begin{cases} ax, & -2 < x < 0 \\ bx, & 0 \leq x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the constants a and b , provided that $EX = \frac{1}{3}$. (6%)

(b) Find the distribution function F_X of X . (6%)

3) Let X be a random variable having a Binomial distribution with parameters n, p .

That is, $X \sim B(n, p)$.

(a) Prove that $P(X = x) = P(Y = n - x)$, for $x = 0, 1, \dots, n$, where

$$Y \sim B(n, q), \quad q = 1 - p. \quad (6\%)$$

(b) What are the mean and variance of X ? (4%)

4) Let X, Y be two random variables having joint p.d.f. given by

$$f_{X,Y}(x, y) = \begin{cases} \gamma, & 0 < x < y, 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the constant γ . (5%)

(b) Find the p.d.f.'s f_X, f_Y of X, Y , respectively. (8%)

(c) Are X, Y independent? Why? (4%)

5) Evaluate the following limits: (12%)

$$(a) \lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x}) \quad (b) \lim_{x \rightarrow 1} \frac{x^3 - 1}{4x^3 - x - 3} \quad (c) \lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t \, dt.$$

6) Find all local extreme values and inflection points of the function

$$f(x) = 2x^4 - 4x^2 + 1. \quad (10\%)$$

7) Let $f(x, y, z) = ye^{2z} \ln(xy)$. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$. (12%)

8) Evaluate the following integrals:

(a) $\int \frac{xe^{2x}}{(2x+1)^2} dx$. (8%)

(b) $\iint_A xy^2 \, dx \, dy$, where $A = \{(x, y) | x > 0, y > 0, 0 \leq x + 2y < 1\}$. (8%)