淡江大學九十三學年度轉學生招生考試試題 761

系別：統計學系三年級　　科目：機率與微積分

准帶項目請打「○」否則打「×」
×	簡單型計算機

節次：7月14日第3節
本試題共 1 頁

1) (a) Let A, B be two events with probabilities of occurrence $P(A)=\frac{3}{8}$, $P(B)=\frac{3}{4}$, respectively. Prove that $P(A^c \cap B^c) \le \frac{1}{4}$ and $\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$, where for a given event E, E^c is the complement of E. (10%)

(b) Let A, B, C be three events with probabilities of occurrence $P(A)=P(B)=P(C)=1$. Prove that $P(A \cap B)=P(A \cap B \cap C)=1$. (6%)

2) Let X be the random variable denoting the number of a certain item sold by a merchant in a given day. Suppose that the p.d.f. of X is given by

$$f_X(x)=\left(\frac{1}{2}\right)^{x+1}, x=0,1,2,\ldots$$

(a) Calculate the probability that no items are sold. (4%)

(b) Calculate the probability that an odd number of items is sold. (5%).

(c) Calculate the average number of items sold. (5%)

3) Let X, Y be two random variable having joint p.d.f. given by

$$f_{X,Y}(x,y)=\begin{cases} c, & 0<x<y<2 \\ 0, & otherwise \end{cases}$$

(a) Find the constant c. (4%)

(b) Find the marginal p.d.f. of X, Y, respectively. Are X, Y independent? (8%)

(c) Find the covariance of X, Y. (8%)

4) (a) Evaluate the limit $\lim_{x\to+\infty} \frac{e^{ax}}{x^2}$, where α is a constant. (8%)

(b) Given that $\int_{-\infty}^{\infty} e^{-\frac{t^2}{2}}\,dt=\sqrt{2\pi}$. Find $I=\lim_{x\to\infty}\int_0^x e^{-3t^2}\,dt$. (8%)

5) Find all relative extrema and points of inflection of the function

$$f(x)=\frac{1}{4}x^4-2x^2. \quad (10\%)$$

6) If $\log yz + y\log z = x$, find the partial derivatives $\frac{\partial y}{\partial z}$ and $\frac{\partial y}{\partial x}$. (10%)

7) Evaluate the following integrals:

(a) $\int \frac{e^{4x}-e^{2x}+1}{e^x}\,dx$. (6%)

(b) $\iint_A xy\,dx\,dy$, where $A=\{(x,y)\mid x>0, y>0, 1\le x+y<2\}$. (8%)