

淡江大學九十二學年度轉學生招生考試試題

系別：統計學系三年級

科目：機率與微積分

准帶項目請打「○」否則打「×」	
×	簡單型計算機

本試題共 / 頁

1. For the r.v. X with p.d.f. $f(x) = (1/2)^x$, $x=1,2,3,\dots$

- (1) Calculate the $E(X)$ (10%)
 (2) Calculate the probability $P(X \leq 3)$. (5%)

2. The r.v. X has d.f. F given by:

$$F(x) = \begin{cases} 0, & x \leq 0 \\ 2c(x^{\frac{1}{2}} - \frac{1}{3}x^3), & 0 < x \leq 2 \\ 1, & x > 2 \end{cases}$$

- (1) Determine the constant c. (5%)
 (2) Determine the corresponding p.d.f.. (5%)
 (3) Calculate the probability $P(X < 1)$. (5%)

3. The r.v.'s X, Y, and Z have joint p.d.f. given by:

$$f_{X,Y,Z}(x,y,z) = 8xyz, \quad 0 < x < 1, 0 < y < 1, 0 < z < 1.$$

- (1) Derive the marginal p.d.f.'s f_X, f_Y , and f_Z . (10%)
 (2) Show that the r.v.'s X, Y, and Z are independent. (5%)

4. Let the r.v. X be distributed as $U(0,1)$ and set $Y = -\log X$.

- (1) Determine the p.d.f. of Y. (5%)
 (2) Calculate the probability $P(Y > 10)$. (5%)

5. Find the following limits. (15%)

$$(1) \lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^2 - 4} \quad (2) \lim_{x \rightarrow 0} (x \sin \frac{1}{x}) \quad (3) \lim_{x \rightarrow \infty} (1 + \frac{3}{x})^{2x}$$

6. Discuss the continuity of the function below at 3. (10%)

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & \text{if } x \neq 3 \\ 6, & \text{if } x = 3 \end{cases}$$

7. Use implicit differentiation to find dy/dx if:

- (1) $3x + 4y - 5 = 0$ (5%)
 (2) $3x^2 + 4y^2 - 2x = 0$ (5%)

8. Find the following integrals. (10%)

$$(1) \int_4^9 \frac{1}{\sqrt{x+x}} dx \quad (2) \int_0^1 \int_{-y}^y (x^2 + y^2) dx dy$$