淡江大學九十一學年度日間部轉學生招生考試試題

系別：統計學系三年級　　　　科目：機率與微積分

准帶項目請打「○」否則打「×」
計算機
×

本試題雙面印製

1) For any event E, let E^c be the complement of E.

   (a) If $P(A \mid B) = P(A \mid B^c)$, prove that A, B are independent. (5%)

   (b) If $P(A \mid C) > P(B \mid C)\ and\ P(A \mid C^c) > P(B \mid C^c)$, prove that $P(A) > P(B)$. (5%)

2) Let X be a random variable having distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ 2x^2, & 0 \le x < \frac{1}{2} \\ 1-2(1-x)^2, & \frac{1}{2} \le x < \frac{3}{4} \\ 1, & \frac{3}{4} \le x \end{cases}$$

   (a) Find $P\left(\frac{1}{4} < X < \frac{5}{8}\right)$. (4%)

   (b) Find a probability density function (p.d.f.) of X. (4%)

   (c) Find the mean and variance of X. (6%)

3) Let $X\ and\ Y$ be two random variables having joint p.d.f.

$$f(x,y) = \begin{cases} 1, & 0 < x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases}.$$

   Let $Z = X + Y$ and $W = X - Y$.

   (a) Find $P(X < 2Y)$. (4%)

   (b) Find the distribution function and the p.d.f. of Z. (8%)

   (c) Find the joint p.d.f. of $Z\ and\ W$. (8%)

   (d) Find the p.d.f. of W. (6%)

4) (a) What is the p.d.f. of a standard normal random variable? Using a property of a p.d.f.,

   prove that $\int_0^\infty e^{-\frac{x^2}{2}}\, dx = \sqrt{\frac{\pi}{2}}$. (4%)

   (b) Let $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx, \alpha > 0$. Prove, by integrating by parts, that

   $\Gamma(\alpha+1) = \alpha\,\Gamma(\alpha)$. (6%)

   (c) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (Hint: Use the change of variable technique, letting $x = \frac{t^2}{2}$

   in the integral of $\Gamma\left(\frac{1}{2}\right)$) (6%)

淡江大學九十一學年度日間部轉學生招生考試試題

系別：統計學系三年級　　　　科目：機率與微積分

准帶項目請打「○」否則打「×」
計算機
×

5) Find the following limits. (16%)

(a) $\lim_{x \to -\infty} \frac{|x|}{x}$　　(b) $\lim_{x \to 0^+} \left(\frac{1}{1-x}\right)^{-\frac{1}{x^2}}$

(c) $\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{x^2}$　　(d) $\lim_{x \to \infty} \left(\frac{3x+2}{2x}\right)^{5x}$

6) Find the following integrals. (10%)

(a) $\int_0^1 \int_1^{e^y} x^2 \, dx \, dy$　　(b) $\int_1^e \int_0^{\ln x} \frac{y}{x} \, dy \, dx$

7) Let

$$u = 3x + 2y - z$$
$$v = x - y + z$$
$$w = x + 2y - z$$

Find the Jacobian $J = \frac{\partial(x,y,z)}{\partial(u,v,w)}$. (8%)