淡江大學八十九學年度日間部轉學生招生考試試題 68

系別：統計學系三年級　　　　科目：機率與微積分

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本試題雙面印製

(1) If $f(x_1, x_2) = (\frac{2}{3})^{x_1+x_2}(\frac{1}{3})^{2-x_1-x_2}$, $(x_1, x_2) = (0,0), (0,1), (1,0), (1,1)$, zero elsewhere, is the joint p.d.f. of X_1 and X_2, find the joint p.d.f. of $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$.
(10%)

(2) Let X_1 and X_2 be observations of a random sample from a distribution with p.d.f. $f(x) = 2x$, $0 < x < 1$, zero elsewhere. Evaluate the conditional probability $Pr(X_1 < X_2 \mid X_1 < 2X_2)$
(10%)

(3) If $E(X^r) = [(r+1)!](2^r)$, $r = 1, 2, 3, \cdots$, find the m.g.f. and p.d.f. of X.
(10%)

(4) Let $\sigma_1^2 = \sigma_2^2 = \sigma^2$ be the common variance of X_1 and X_2 and let ρ be the correlation coefficient of X_1 and X_2. Show that
(10%)

$$Pr\left[\,\left|(X_1-\mu_1)+(X_2-\mu_2)\right| \geq k\sigma\right] \leq \frac{2(1+\rho)}{k^2}.$$

(5) Given the distribution function
(10%)

$$F(x) = \begin{cases} 0, & x < -1, \\ \dfrac{x+2}{4}, & -1 \leq x < 1, \\ 1, & 1 \leq x, \end{cases}$$

then compute : (a) $Pr(-\frac{1}{2} < X \leq \frac{1}{2})$; (b) $Pr(X=0)$; (c) $Pr(X=1)$;
(d) $Pr(2 < X \leq 3)$

(6) Find the following limits :
(10%)

(a) $\lim\limits_{x\to 0}(x+e^x)^{\frac{2}{x}}$; (b) $\lim\limits_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$; (c) $\lim\limits_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos 2x}$;

(d) $\lim\limits_{x\to 0}\frac{2x+1-\cos x}{3x}$ (e) $\lim\limits_{x\to 0^+}\frac{\cot x}{\ln x}$.

(7) 利用均值定理証明
(10%)

$$1.5 < \sqrt{3} < 1.75.$$

◀注意背面尚有試題▶

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(8) Find (a) $\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{3/2}}\,dx$; (b) $\int_0^1 \tan^{-1}x\,dx$

(10%)

(9) If $\sinh xy = ye^x$, then $y' = \frac{dy}{dx} = ?$

(10%)

(10) Test for convergence :

(10%) (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$, (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+2n+2}$, (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n-1}$